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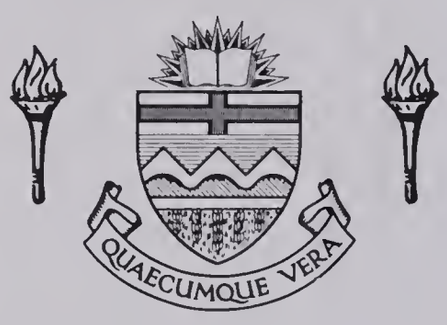

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A STAND GROWTH MODEL FOR TREMBLING ASPEN IN THE PRAIRIE PROVINCES OF CANADA

by

© T.I.J. GRABOWSKI

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

IN

FOREST SCIENCE

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FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled A Stand Growth Model for Trembling Aspen in the Prairie Provinces of Canada submitted by T.I.J. Grabowski in partial fulfilment of the requirements for the degree of Master of Science in Forestry.

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Two undergraduate research assistants, Sandy Fordyce and Bob Merta, worked steadfastly on field data collection during the summer of 1980. Sandy continued to assist with the analysis during the remainder of the project on a part time basis. Even when faced with the most tedious of tasks, such as data entry and processing, Sandy performed with diligence and intelligence. Also contributing greatly to the data entry effort for a portion of the summer of 1980 was Heather Jamha.

On a personal note, the author wishes to express gratitude to his parents, who instilled in him from a very young age the desire for knowledge and wisdom. Finally, the author thanks his Lord and Saviour Jesus Christ, who gave him the endurance to see this project through to its completion.

"If any of you lacks wisdom, let him ask God, who gives to all men generously and without reproaching, and it will be given him."

James 1:5

ABSTRACT

An individual tree/distance independent growth model was constructed for trembling aspen in the prairie provinces of Canada. Five basic relationships are integral to the model; mortality, diameter growth, height growth and site differentiation, volume estimation, and biomass estimation. The first two relationships were developed specifically for this study; the latter three were obtained from the literature. The aspen growth model provides summaries of stand characteristics by diameter class at selected ages for the simulated stand. Testing and evaluation indicated satisfactory model performance and significant advantages for the model structure chosen.

GLOSSARY

1. DBH: Tree diameter at breast height (1.3 metres above ground level).
2. BA: Tree basal area, usually at breast height.
3. SI: Site index; a measure of site quality; the average height of dominant and co-dominant trees in a stand at some reference age (usually 50 years for trembling aspen).
4. MAI: Mean annual increment of a tree or stand parameter such as biomass.

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1. INTRODUCTION

The objective of this study was to develop a forest stand growth model, primarily for the prediction of biomass and volume growth, for trembling aspen (*Populus tremuloides* Michx.) in the prairie provinces of Canada. The basic purpose for the development of a forest stand growth model is to allow managers and researchers to simulate the growth and development of forest stands under a variety of natural and artificial conditions. Simulation models, such as the one described in this thesis, permit evaluation of forest management alternatives in a quick and inexpensive manner.

1.1 Growth Models

Forest stand growth models have been classified into three types: single tree/distance dependent, single tree/distance independent and whole stand/distance independent (Hann 1979, Munro 1974). Models in the first category project growth on an individual tree basis for all trees in the stand. Locational data for each tree in the stand are required to evaluate individual competitive status. Examples of this type include Bella (1970), and Newnham and Smith (1964).

Models of the single tree/distance independent category do not require tree data on location of trees. The growth of individual trees in the stand is projected and competition is determined through a comparison of the characteristics of

an individual tree with the average stand characteristics (Hann 1979, Munro 1974). Included in this category are Stage's (1973) *Prognosis Model* and the U.S.D.A. Forest Service (1979) *Generalized Forest Growth Projection System.*

Models from the last category, whole stand/distance independent, project growth on a stand level basis, and provide information about average stand conditions (Hann 1979, Munro 1974). A variety of different types of whole stand models exist within this category. These range from models which neither use nor provide diameter distribution data (Pienaar and Turnbull 1973, Moser 1972, and Moser and Hall 1969) to those utilizing diameter distribution functions (Clutter and Belcher 1978), to those using diameter class information (Ek 1974, Peden et al. 1973).

Conceptually, it may be clearer to view forest growth models as existing on a continuum ranging from single tree/distance dependent to whole stand/distance independent. These categories represent marginal differences between various models on the continuum. For example, there may be no difference between a single tree/distance independent model which measures tree diameters to the nearest 2mm., and a whole stand/distance independent model which divides trees into 2mm. classes and projects their growth as such. As one moves along the continuum away from the single tree/distance dependent models, the informational requirements decrease as do (presumably) the predictive abilities of the models (Hann 1979). Ek and Dudek (1980) and Hann (1979), however,

question the trend in predictive ability. In particular, the superiority of single tree/distance dependent models over single tree/distance independent models has been questioned (Ek and Dudek 1980).

1.2 Approach to Model Development

The individual tree/distance independent format was chosen for this study for a number of reasons. First, previous attempts by the Department of Forest Science to model the growth of trembling aspen using a whole stand model provided unsatisfactory simulation results and a general lack of model flexibility. Second, the superiority of individual tree/distance dependent models has not been proven and the data requirements for the development of this type of model were prohibitive in terms of both the physical and financial limits for this study. Finally, it was felt that an individual tree/distance independent model would allow sufficient flexibility for model application, would improve previous growth predictions, and could be developed from available data and from data which could be collected within a realistic time span.

The model for this thesis (hereafter referred to as *'the aspen growth model'*)was developed in five stages. The first stage consisted of a review of the available literature on forest growth modelling and aspen stand development. Findings of the literature review of forest growth modelling are presented in this introduction and

influenced the decision to construct an individual tree/distance independent model. In the second part of the literature review the biological concepts underlying aspen growth and development were addressed as they pertain to model construction. An attempt was made to identify the factors that affect aspen growth and should, therefore, be incorporated in a realistic growth model.

The second stage in the development of the aspen growth model was to assess the extent and quality of existing growth data for aspen from various government agencies and forest products companies in the prairie provinces of Canada, and to obtain those data which appeared useful for model development. Age, density and geographic deficiencies in this data base were identified and supplementary field sampling was conducted to augment the existing data.

The third stage addressed the specification and evaluation of the basic model relationships. Five basic relationships were identified as integral to the aspen growth model:

1) Mortality,
2) Diameter growth,
3) Height growth and site differentiation,
4) Volume estimation,
5) Biomass estimation.

An investigation of these relationships indicated that the latter three could be obtained from the literature. The first two relationships, mortality and diameter growth, are

key elements to the aspen growth model and were developed and tested specifically for this study.

After the five basic relationships had been procured or developed, they were assembled, in the fourth stage, into an individual tree/distance independent model framework. The resulting aspen growth model accepts a vector of individual tree diameters as input and then projects diameter growth, height growth and mortality. Various stand parameters such as biomass, volume, basal area and density on a per hectare basis are are then summarized by diameter class. Also included in the stand summary are average stand parameters such as diameter at breast height (DBH), stand height, and mean annual increment.

Finally, the model was tested and evaluated using data from representative permanent plots. Growth of three sample plots was simulated for a period of 25 years and the model results compared to actual plot values. These plots had been used in a portion of the development of the aspen growth model, so the tests were not totally independent (see '**MODEL TESTING AND EVALUATION**').

The structure of this thesis closely follows the outline presented above for the development of the aspen growth model. A literature review of aspen stand development, from regeneration to juvenile stands to mature and overmature stands, follows this introduction. Data collection efforts are described, followed by a discussion of the selection, or development and testing, of each of the

five basic model relationships. Details of model construction followed by model testing and evaluation are then described. A concluding chapter includes both general statements about the accuracy and adequacy of the aspen growth model and suggestions for additional research.

2. LITERATURE REVIEW - ASPEN STAND DEVELOPMENT

An essential element in the construction of the aspen growth model was a review of the available literature describing growth and development of aspen stands. This discussion of the life cycle of aspen is divided into three parts: 1) Regeneration and early years; 2) Juvenile stands; and 3) Mature and overmature stands. Factors which are considered important to model development are given the greatest attention in this review.

2.1 Regeneration and Early Years (≤ 10 Years)

2.1.1 The Clonal Concept

Trembling aspen occurs throughout most of Canada and the northern and western United States as a clonal species (Steneker 1976, Copony and Barnes 1974, Steneker and Wall 1970, Barnes 1969 and 1966, Zahner and Crawford 1965, Basham 1958). Each clone consists of genetically identical individuals (ramets) which have arisen from the root system of a single parent (ortet) (Barnes 1966). Ramets within a clone share similar phenotypic and silvical characteristics (Lehn 1979, French and Manion 1975, Copony and Barnes 1974, Schier 1974, Schier and Johnston 1971, Barnes 1969 and 1966, Zahner and Crawford 1965).

Inter-clonal differences occur for many traits including time of flowering, time of leaf flushing and leaf fall, leaf and twig characteristics, bark color and texture,

and tree form (Kemperman 1977, Barnes 1975, Steneker and Wall 1970). Lehn (1979) reported inter-clonal differences in biomass production. Height differences between clones of the same age have been reported but there is some question as to their magnitude (Jones and Trujillo 1975, Zahner and Crawford 1965). Inter-clonal differences in susceptibility to frost damage, *Hypoxylon pruinatum* canker and decay organisms seem to be prominent (Kemperman et al. 1978, French and Manion 1975, Copony and Barnes 1974, Wall 1971 and 1969, Egeberg 1963). Suckering ability also appears to be highly related to clonal differences (Schier 1976 and 1974, Zasada and Schier 1973, Garrett and Zahner 1964).

2.1.2 Clonal Effects on Aspen Suckering

Although some aspen are established by seed (McDonough 1979, Andrejak and Barnes 1969, Barnes 1969), most regeneration is vegetative (Barnes 1969 and 1966). Garrett and Zahner (1964) reported that clonal variation in suckering intensity was so great that the effects of various clearcutting techniques were obscured. Significant clonal differences have also been found in the number and dry weight of suckers produced from root cuttings, as well as in the content of total nonstructural carbohydrates (Schier 1976 and 1974, Zasada and Schier 1973, Schier and Johnston 1971). Suckers from different clones respond differently to chemical treatments (Schier 1976, 1973a, 1973c) and to different temperature treatments (Schier 1976, Zasada and

Schier 1973, Maini 1967). Clonal differences in the disturbance requirements for aspen suckering may also exist (Schier 1976). All of these differences are probably modified by date of root collection (Schier 1973d), site conditions and other non-genetic characteristics.

Barnes (1966) reported that clones become established and interact with each other depending on ease of seedling establishment, rate of root expansion, inherent suckering ability (i.e. genetic differences between clones) and the amount of disturbance. Very little is known, however, about clonal dynamics in a population. Clones expand, contract and establish themselves in a stand but the effects of the competitive nature of clones throughout their life span is not documented in the literature.

Ideally, a forest growth model for trembling aspen should include significant clonal effects on growth and mortality. However, since no growth data on a clonal basis is available for aspen, the growth model described here does not account for any clonal differences in stand characteristics.

2.1.3 Apical Dominance Phenomenon and Temperature Effects

Aspen suckers arise from newly initiated meristems on aspen roots after the cork cambium has been formed. These buds may develop into shoots or remain at the primordial stage where later they may develop in response to other stimuli (Schier 1976).

It appears that sucker formation is primarily suppressed by auxin transported from growing shoots (Farmer 1962; Schier 1973a, 1975 and 1976, Steneker 1974). This apical dominance must be disrupted in order for suckering to occur. Interference with auxin control may occur as a result of girdling, decapitation, defoliation or removal of shoot tips (Schier 1976). Application of an anti-auxin or severe disturbances such as fire or clearcutting may also induce suckering (Schier 1975). Drastic disturbances, however, are not necessary for sucker production as is evidenced by suckering in undisturbed aspen stands (Schier 1973b). Environmental changes during normal seasonal growth may disrupt apical dominance enough to allow suckering to occur (Schier 1976).

Once the apical dominance effect has been broken, environmental factors become important (Steneker 1974). Soil temperature is probably the most important of these environmental influences (Maini and Horton 1966, Steneker 1974, Steneker and Walters 1971, Zasada and Schier 1973). Low soil temperatures may be a limiting factor in Alaska where aspen is found chiefly on southern exposures (Zasada and Schier 1973). High soil temperatures on the edges of aspen stands may account for aspen invasion into grassland areas (Barley and Wroe 1974). Sucker frequency and height vary inversely with the depth and diameter of the parent roots, probably indicative of the effects of soil temperature and carbohydrate reserves (Kemperman 1978).

An aspen sucker is dependent upon carbohydrate reserves in the parent root until it can carry on adequate photosynthesis by itself (Schier 1976, Schier and Zasada 1973). The number of suckers produced from an aspen root (i.e. not necessarily emerging from the soil surface) is not limited by carbohydrate reserves. The density of suckers that reach the soil surface, however, is related to these reserves since sucker growth through the soil is related to the level of carbohydrate reserves in the parent root (Schier 1976). Depletion of root reserves by repeated disturbances such as cutting, burning or insect attack can severely reduce sucker production (Berry and Stiell 1978, Schier 1976). This potential for a reduction of reproductive capacity would be an important consideration if components for simulating harvesting, insect attack, etc. were incorporated into the aspen growth model. In its initial development phase, however, the aspen growth model does not have such routines. It was designed to operate on a single rotation basis, with a structure that would easily allow later introduction of components simulating natural and man-caused changes.

2.1.4 Factors Affecting Suckering After Harvest or Fire

Aspen suckers prolifically, (up to 200,000 per ha.), following a severe disturbance such as fire or clearcutting (Schier and Smith 1979, Kemperman 1978, DeByle 1976, Steneker 1976, Bella and DeFranceschi 1972). After logging

the density of suckers produced is proportional to the number of stems removed, with the best regeneration occurring after a total clearcut (Schier and Smith 1979, Bella and De Franceschi 1972). As little as 2.3 to 3.4 square meters per hectare of basal area of residual overstory can significantly reduce suckering (Perala 1977). This is another factor to be considered if a harvesting option were incorporated into the aspen growth model. It is not clear whether summer or winter logging results in the greatest sucker density (Bella and DeFranceschi 1972, Stoeckler and Macon 1956, Sandberg and Schneider 1953) although some reports suggest that suckering should be greatest on summer cutblocks due to increased site disturbance and destruction of both understory and humus resulting in higher soil temperatures (DeByle 1976, Bella and DeFranceschi 1972). Disking has been shown to stimulate sucker production (Zillgitt 1951) but scarification has no apparent effect (Schier and Smith 1979).

Fire, girdling or trenching can also promote aspen suckering by disrupting the apical dominance effect (Schier and Smith 1979, Steneker 1974, Buckman and Blankenship 1965, Farmer 1962, Stoeckler 1948). Repetition of intense disturbances such as repeated spring burning or very short rotation lengths can detrimentally affect suckering, presumably due to a depletion of root reserves in the parent tree (Berry and Stiell 1979, Buckman and Blankenship 1965).

2.1.5 Sucker - Density Relations

During the first few years of life many suckers die due to suppression or pathological factors (Bella 1975, Bella and DeFranceschi 1972, Anderson and Anderson 1968). Grazing by cattle or wildlife can also cause significant mortality (DeByle 1976). Aspen sucker stands show a marked approach towards a common density level, with very dense stands suffering the highest mortality and the least dense stands suffering little or no mortality (Sorenson 1968, Strothmann and Heinselman 1957). In one study (Bella and DeFranceschi 1972), sucker stands with densities ranging from 17 to 240 thousand stems/ha. at one year of age all approached a density level between 22 and 53 thousand stems/ha. by the age of six. Concerns about harvesting techniques could, therefore, be partially nullified due to this trend towards the elimination of initial differences in density. These highly variable mortality rates presented a significant problem and, therefore, required special attention in the development of the mortality component of the aspen growth model (see '**BASIC MODEL RELATIONSHIPS - Mortality**').

Diameter and height growth of aspen appear to be affected by stand density in the same manner as most other tree species. Diameter growth decreases with increased stand density (Bella 1975); therefore, stand density, or some transformation thereof, is a logical variable for inclusion into the diameter growth component of the aspen growth model (see '**BASIC MODEL RELATIONSHIPS - Diameter Growth**'). Height

growth does not appear to be greatly affected by stand density and shows a remarkable constancy over a wide range of densities (Bella 1970, Strothmann and Heinselmann 1957). For this reason, a simple height growth function predicting height on the basis of site quality and tree age and ommitting independent variables such as dbh and density is logical for the aspen growth model (see '**BASIC MODEL RELATIONSHIPS - Height Estimation and Site Differentiation**').

2.2 Juvenile Stands (10≤ Age ≤40 Years)

2.2.1 Density Reduction in Sapling Stands Due to Hypoxylon Canker

Mortality in young stands of trembling aspen is usually caused by suppression or disease (Hinds and Krebill 1975, Anderson and Anderson 1968). Hypoxylon canker is caused by the fungus *Hypoxylon pruinatum* (Klotz.) Cke. and is one of the most important pathological agents throughout the natural range of trembling aspen (Anderson and Anderson 1968, Baranyay 1967, Gruenhagen 1945, Povah 1924). The fungus kills its host by rapid invasion and subsequent girdling of the stem tissues (Anderson and Anderson 1968, Povah 1924). Mycelia of *H. pruinatum* penetrate the bark, cambium and outer four to five mm. of wood tissue (Gruenhagen 1945). There does not appear to be any significant reduction in basal area increment of infected trees during the infection period, although the yearly

increment is re-allocated away from the canker producing an off-centered growth pattern (Baranyay 1967).

Hypoxylon cankers usually form only on young bark (Bier 1940). As a consequence, large trees incur infection on branches or on the bole near the crown while young trees become infected on the bole near the ground (Brinkman and Roe 1975, Anderson and Anderson 1968, Gruenhagen 1945, Povah 1924). Infection on the bole will usually result in mortality in three to eight years (Anderson and Anderson 1968, Baranyay 1967).

Graham and Harrison (1954) found Hypoxylon canker in the Lake States to be associated with wounds inflicted by insects. Baranyay (1967) found that dead branch stubs were the main port of entry for Hypoxylon canker in Alberta. Other channels of entry may be provided by such items as wounds caused by wildfires, lightening, frost cracks, sun scald, animal browsing and human activities (Hinds and Krebill 1975, Gruenhagen 1945).

There appears to be an inverse relationship between the density of aspen stands and the incidence of Hypoxylon canker (Anderson and Anderson 1968, Day and Strong 1959, R.L. Anderson 1952), although there is some doubt about this suggested relationship (Steneker 1966). Copony and Barnes (1974) confirmed this relationship, but suggested that mortality caused by Hypoxylon canker was the reason for low stand density. This density relationship could have important ramifications on thinning strategies in areas

where Hypoxylon canker is prevalent (Anderson and Anderson 1968).

The incidence of Hypoxylon canker in an aspen stand is not related directly to site quality, vigor or sex of the tree (Brinkman and Roe 1975, Anderson and Anderson 1968, R.L. Anderson 1964, G.W. Anderson 1958, Christensen et al. 1951). Clones with superior growth, therefore, are not necessarily the most resistant. There does appear to be a relationship, however, between Hypoxylon canker incidence and geographic location (R.L.Anderson 1964).

There is significant inter-clonal variation in the incidence of Hypoxylon canker (French and Manion 1975, Copony and Barnes 1974). Copony and Barnes (1974) reported eight-fold differences in Hypoxylon canker infection between clones (ten percent infection versus eighty percent infection). Selection of "good" clones, therefore, should take Hypoxylon canker into account.

No direct control measures are known for Hypoxylon canker (Brinkman and Roe 1975, Hinds and Krebill 1975). Maintenance of well-stocked stands throughout the rotation seems to be the best approach (Brinkman and Roe 1975, Anderson and Anderson 1968).

Hypoxylon canker infection or widespread outbreak is another event with potential for inclusion into the aspen growth model. Quantifying the relationships between Hypoxylon canker and stand density or other variables would be necessary before a Hypoxylon infection routine could be

developed.

2.2.2 Effects of Thinning

Decreasing the density of an aspen stand by thinning generally results in an increase in radial growth of individual trees (Brinkman and Roe 1975, Bella 1970, Jarvis et. al. 1966). Aspen trees in all diameter classes will respond to thinning, with the smaller trees showing the greatest percent diameter increment relative to the initial tree diameter (Bella 1970). The larger trees, however, have a higher increase in growth rates in absolute terms than smaller trees (Bella 1970). The loss in volume of the thinned trees appears to offset changes in diameter growth so that stand increment and total volume yield is often not increased by thinning (Brinkman and Roe 1975, Bella 1970). Therefore, thinning does not appear to be a viable management option and for this reason was not considered important for the aspen growth model.

2.3 Mature and Overmature Stands (≥40 Years)

2.3.1 Decay and Mortality

Mature and overmature aspen stands are characterized by a slowdown in radial growth and an increase in decay volume (Kirby 1962, Basham 1960, Kirby et al. 1957, Kaufert 1948). While stands of fully stocked, mature aspen may be comparable to softwood species in gross volume, their commercially usable volumes are much smaller due to decay.

This characteristic has led to the term "pathological rotation", which is the rotation age based on net volume increment, taking into account the effects of decay (Kirby et.al. 1957). The intended utilization of aspen greatly affects the selection of rotation age since some processes, such as biomass production for energy purposes, may be able to use different parts of the aspen tree (eg. small branches) and may also be able to use decayed wood.

Fomes igniarius var. *populinus* (Neuman) Campbell (referred to throughout this report as *F.igniarius*), a heart rot, is the most serious agent of decay in trembling aspen in North America (Anderson and Schipper 1978, Kirby et al. 1957, Riley 1952, Christensen et al. 1951, Kaufert 1948). This decay organism enters the aspen trunk primarily through branch stubs and wounds (Kirby et al. 1957, Christensen et al. 1951). An average of six years is required between the time of infection and the appearance of the first fruiting bodies, or conks (Anderson and Schipper 1978). Conks and other external indicators such as visible wounds have been used to predict the presence and amount of internal decay in aspen trees with varying degrees of success (Anderson and Schipper 1978, Bailey and Dobie 1977, Alemdag and Horton 1972, Wall 1969, Basham 1968, Hinds 1963, Riley and Bier 1936). Mortality from *F. igniarius* appears to be primarily the result of structural weakening of the bole and subsequent blowdown (Anderson and Schipper 1978).

Decay in trembling aspen has often been referred to

simply as "cull" or "defect" but should be divided into classes in order to evaluate the seriousness of the loss and its importance to different uses. Kemperman et.al. (1978) classify wood defect as "advanced rot", "incipient rot", and "stain". Advanced rot is noticably soft; incipient rot is discolored wood that is "slightly softer" than clear sound wood when tested with the point of a sharp knife; and stain is discolored wood that is as hard as clear, sound wood. It is important to note that although defect volumes for aspen can exceed 30% of total gross volume (Hinds and Wengert 1977, Kirby et.al. 1957) , much of this defect is probably stain or incipient decay. It has been found that advanced rot is by far the smallest component of total defect (Kemperman et.al. 1978, Kemperman et.al. 1976, Navratil 1972).

Several variables such as site index, soil, and aspect have been examined for possible relationships with the incidence and volume of defect, but correlations have been low (Anderson and Schipper 1978, Kemperman et.al. 1978, Kemperman et.al. 1976, Thomas et.al. 1960, Basham 1958, Riley 1952). This could be due to the effect of genotype on defect. Highly significant differences have been found among clones in the incidence of *F.igniarius*, percent decay, volume of decay, gross volume and net volume (Kemperman et.al. 1978, Wall 1971 and 1969). In one study (Wall 1969), incidence of *F.igniarius* among intermingled clones on the same site varied from 12 to 64%; on another site the

variation was from 21 to 92%. Kemperman, et al. (1978) found significant differences between clones in the volume of stain and incipient decay but not in advanced decay. They also found that the primary rot causing organism varied by clone. In some cases *F.igniarius* was the primary causal agent, while in others rot was caused primarily by *Radulum caeserium* (Morg.)Lloyd or various other butt rot fungi. If the decay relationships were quantified and related to other variables, then a decay estimation routine might be an important option to the aspen growth model.

2.3.2 Forest Tent Caterpillar

The forest tent caterpillar, *Malacosoma disstria* Hbn., is the most spectacular defoliator of aspen in the prairie provinces (Steneker 1976, Ives 1971). Epidemic populations can develop in the same areas at 10- to 15-year intervals (Brinkman and Roe 1975). Ives (1971) reviewed the outbreak history of forest tent caterpillar in Alberta.

Radial growth of aspen trees can decrease as much as 80 to 90 percent after several years of severe defoliation and in general is reduced with increasing intensity and frequency of defoliation (Brinkman and Roe 1975, Ives 1971, Duncan and Hodson 1958). Few trees die following forest tent caterpillar infestations (Brinkman and Roe 1975, Ives 1974) and growth returns to normal within two years after severe defoliation (Ives 1971, Duncan and Hodson 1958). Forest tent caterpillar infestations could also be modelled if the

relationship between caterpillar attack and tree growth were further quantified. Although forest tent caterpillar can be controlled using pesticides, this rarely happens because of the relatively low economic importance of aspen in the prairies (Brinkman and Roe 1975).

2.3.3 Other Insects

Other insects which can cause significant damage to trembling aspen on the prairies include the large aspen tortrix, *Choristoneura conflictana* Wlk., the gray willow-leaf beetle, *Galerucella decora* Say, the aspen leaf beetle, *Chrysomela crotchi* Brown, and the poplar borer, *Saperda calcarator* Say (Davidson and Prentice 1968). At the present time the control of these insects using pesticides is unwarranted given the low level of aspen utilization on the prairies. This management option may be employed, however, at some time in the future (Brinkman and Roe 1975, Davidson and Prentice 1968).

2.3.4 Site Relationships

Trembling aspen can inhabit a wide range of sites but grows best on fresh to moist clay loams and moist sandy loams that have good drainage and a water table between .75 and 2.5 meters below the surface (Steneker 1976, Fralish and Loucks 1967). Stoecklер (1960) found that repeated burns lowered site quality, but Strothmann (1960) found no such relationship and later publications (Steneker 1976, Fralish

and Loucks 1967) do not even mention this factor.

Several systems have been developed for the rating of aspen site quality or aspen site index from site characteristics such as soil texture, soil moisture, stone content, slope, aspect, and depth to water table (Steneker 1976, Brinkman and Roe 1975, Fralish and Loucks 1967, Strothmann 1960, Meyer 1956) but site index calculated as a function of the average heights of the dominant and co-dominant trees is still the most commonly used measure of aspen site quality, and is, therefore, used in the aspen growth model.

2.3.5 Summary

In conclusion, it has been shown that the dynamics of aspen stand development are very complex. A number of potential options for an aspen growth model are evident including components for the simulation of clonal differentiation, harvesting, fire, disease, insects and other natural or man-caused changes. The aspen growth model was designed to operate on a single rotation basis, with a structure that permits the incorporation of the above-mentioned options, if they are developed. The effects of certain factors are implicitly present in the data which was available for this study, and are, therefore, implicitly present in the aspen growth model. Insect and disease infestations, such as tent caterpillar or Hypoxylon canker outbreaks, likely affected some or all of the stands where

permanent or temporary sample plots were measured. Model relationships developed from these data would, therefore, include the effects of such infestations. This would be an important factor for consideration if components for some of these options were developed.

Several important relationships directly affected the construction of the aspen growth model. The most important of these is the occurrence of extremely variable mortality rates in young aspen stands. In addition, findings of the literature review regarding diameter growth, height growth and site quality estimation were found to be of importance and were subsequently considered when the basic model relationships were developed (see **'BASIC MODEL RELATIONSHIPS'**).

3. DATA COLLECTION

Data were required for the development of two model relationships; diameter growth and mortality (see '**BASIC MODEL RELATIONSHIPS**'). The diameter growth component could be developed from repeated diameter measurements over time of trees in permanent sample plots or from radial increment measurements of trees at a given point in time using an increment borer. In order to indicate mortality, repeated measurements over time of trees in permanent sample plots were required for the development of the mortality component. Both types of data were requested from the following agencies and companies:

1) Provincial forest management agencies:
   - Manitoba
   - Saskatchewan
   - Alberta
   - British Columbia,
2) The Canadian Forestry Service (Northern Forest Research Center, Edmonton),
3) MacMillan Bloedel, Aspenite Division, Hudson Bay, Saskatchewan, and
4) Procter and Gamble Cellulose Ltd., Grande Prairie, Alberta.

Two of these sources eventually proved to have valuable data; the Alberta Forest Service (A.F.S.) and the Canadian Forestry Service (C.F.S.).

The Alberta Forest Service made all of its permanent sample plot data available, 26 permanent sample plot groups in pure aspen types in Alberta (Figure 1, Appendix I). Each group consists of four plots which are either 0.25 or 0.50 acres in size. In total, the Alberta Forest Service permanent plot data consisted of approximately 12,000 individual tree observations (the measurement of an individual tree at a point in time was considered an "observation"). All Alberta Forest Service permanent sample plots included individual tree measurements for diameter at breast height (outside bark) and crown class. In addition, some sample trees had measurements of age, height, bark thickness and radial increments. A majority of these plots were established in 1960 with remeasurements made on all plots in 1968 and on some plots in 1978 and 1979. Four groups of plots which had been remeasured last in 1968 were remeasured again in the summer of 1980 as a part of this study. All of the Alberta Forest Service permanent sample plots were in stands 50 years and older (see the age-density summary, Table 1, Appendix I).

The Canadian Forestry Service also made its permanent sample plots in pure aspen types available, one plot from study MS-133 (Turtle Mountain Forest Reserve, Manitoba); three plots from study MS-155 (Pelly, Saskatchewan); and three plots from study MS-146 (Riding Mountain National Park, Manitoba). These plots were established in age classes ranging from 11 to 23 years and were remeasured by the

Canadian Forestry Service five times at approximately five year intervals. Measurements included diameter at breast height (outside bark) and heights of some sample trees. The Canadian Forestry Service permanent sample plot data were composed of approximately 8,000 individual tree observations.

Analysis of the data from the Alberta Forest Service and the Canadian Forestry Service resulted in a number of conclusions pertaining to model development and data collection efforts to be carried out. First, the mortality component of the model would have to be developed from these data, since these were the only available permanent sample plot records in aspen types. Second, it was decided that supplementary measurement of temporary sample plots was necessary in young, dense stands in Alberta in order to fill geographic, age and density gaps in the data base. Twenty three temporary sample plots were subsequently established in pure aspen types in various Alberta locations (Figure 2, Appendix I) covering a selected range of density and age classes (Table 2, Appendix I).

All of the temporary plots were 0.01 hectares in size. Measurements included species; diameter at breast height (outside bark); total height; crown class; cull suspect class; total age; stump diameter; ten, twenty and thirty year radial increment; and an approximate location of each tree on the plot. Additional data from the remeasurement of some University of Alberta permanent sample plots in pure

aspen types, conducted under another research project in the Department of Forest Science, were also made available to this project.

Several of the other agencies and companies which were contacted made data available for this project but, for a variety of reasons, these data were not used in model development. The Forest Inventory Branch of the Manitoba Department of Natural Resources made inventory data in aspen types available for this project. These data could not be used due to a lack of radial increment measurements.

The Forestry Branch of the Saskatchewan Department of Tourism and Renewable Resources made all of their permanent sample plot data in pure aspen types available for this project. These plots were established in 1949 and 1954 with subsequent remeasurments at approximately five year intervals. All measurements before 1969 were on a diameter class plot tally basis. This prohibited their use for both the mortality component and the diameter growth component of the aspen growth model. In 1969 all trees on these plots were assigned individual numbers and their diameters were recorded individually. Remeasurements on an individual tree basis was scheduled for the summer of 1980. It was expected that the 1969 measurements could be used in conjunction with the 1980 remeasurements for both the mortality and diameter growth components of the aspen growth model. The Saskatchewan Forestry Branch, however, encountered difficulties when remeasuring these plots in 1980 and were

forced to renumber the individual trees on the plots. This made their data unusable.

The remaining agency (British Columbia Forest Service) and the two companies contacted (Procter and Gamble Cellulose, and MacMillan Bloedel, Aspenite Division) were in the process of establishing permanent sample plots in aspen types but had performed no remeasurements on their plots.

4. BASIC MODEL RELATIONSHIPS

Five basic relationships were identified as integral to the development of the aspen growth model:

1) Mortality,
2) Diameter growth,
3) Height growth and site differentiation,
4) Volume estimation,
5) Biomass estimation.

The first two relationships, mortality and diameter growth, are key components of the aspen growth model. No functions were found in the literature which would adequately represent these relationships for aspen. They were, therefore, developed and tested specifically for the aspen growth model. The sections describing the mortality and diameter growth components include discussions of the data from which they were developed, the various approaches utilized to develop the functional forms, and testing of the final functional relationships.

The last three relationships were obtained from the literature in a format which was considered adequate for the aspen growth model. The sources of these relationships, as well as their functional forms, are described in their corresponding sections.

4.1 Mortality

Trembling aspen exhibits extremely high mortality in young, dense stands, with rapid disappearance of initial differences in density (Bella and De Franceschi, 1972). Very dense stands suffer the highest mortality while the least dense stands suffer little mortality (Sorensen 1968, Strothmann and Heinselman 1957). It was felt that the mortality component in young aspen stands was an extremely important aspect of the aspen growth model due to the significant and often drastic effects that mortality may have on certain growth parameters such as stand volume and biomass. Therefore, the mortality component of the aspen growth model was addressed with special attention.

There are two basic methods of modelling mortality for individual tree growth models (Ek and Dudek 1980). In the first, trees are classified into live and dead categories based on some threshold such as a minimum level of growth (Ek and Dudek 1980). One problem with this approach is the arbitrary selection of a threshold parameter and its level. Another problem is that mortality in forests may be caused by numerous factors and is not solely restricted to the smallest, slowest growing trees (Ek and Dudek 1980). An examination of the permanent sample plot data revealed that in young aspen stands the smallest trees are more likely to die, but in older stands there was no obvious relationship between any simple tree parameter and mortality. The second method was established by Hamilton (1974,1980) and Hamilton

and Edwards (1976) and is based on predicting the probability that a tree dies. In this method, the mortality model is developed in three basic steps (Hamilton 1974, 1980, Hamilton and Edwards 1976, Hamilton and Wendt 1975). In the first step potential independent variables are screened for those which best explain variations in mortality. In the second step coefficients are estimated for the logistic model using a dichotomous dependent variable (0=live tree, 1=dead tree). The function form for the logistic model is:

$$P_i = \frac{1}{1 + \exp\left(-(B_0 + B_1 X_{1i} + B_2 X_{2i} \ldots + B_p X_{pi})\right)}$$

Where: P_i =probability of mortality of the ith tree in a specified period of time, restricted to the interval (0,1).

B_j =jth nonlinear regression coefficient, j=0,1...p

X_k =kth independent variable, k=1,2,...p

The probability of mortality for an individual tree is bounded by 0 and 1. Although a linear regression model has been used for the prediction of the probability of tree mortality, it cannot ensure that the probability estimates will lie between 0 and 1 (Hamilton and Edwards 1976). The logistic function, which limits estimates of probability to the interval (0,1), has been proposed as the preferred model for expressing this relationship between the dichotomous

dependent variable (mortality) and the independent variable(s). The final step is model verification by comparison with an independent data set to ensure that it adequately estimates mortality and does not merely reflect anomalies of the particular set of data from which the model was developed. A chi-square test is often used to measure this "goodness of fit" (Hamilton and Edwards 1976).

This method was adopted for the aspen growth model for the following reasons. First, this type of model allows great flexibility in application. The resulting probability of mortality may be used in either a stochastic or a deterministic fashion to predict stand mortality. With the stochastic method, a uniform (0,1) random variate is generated for each tree in the stand, for each time period. The tree "dies" and is removed from the stand if the estimated probability of death is greater than the random variate, otherwise it "lives" (Ek and Dudek 1980, Hamilton and Edwards 1976). The deterministic method treats each tree in the stand as representative of a group of trees (for example, plot tallies expanded to a per hectare basis). The probability of mortality for each tree is then used in a deterministic fashion to reduce the number of trees which it represents (Ek and Dudek 1980, Hamilton and Edwards 1976). The second reason for selecting this model is that a number of potential independent variables are considered for inclusion into the model with the best predictors of mortality being accepted. In this way, the arbitrary

selection of a threshold variable and minimum threshold levels is avoided.

The logistic nonlinear regression package of the Biomedical Computer Programs - P Series (BMDP-79, Dixon and Brown 1979) can be used for screening independent variables for their predictive ability and for estimating the appropriate coefficients for the logistic function. Using a subset of 500 tree observations from the A.F.S. and the C.F.S. permanent sample plot records for both young and old stands, the following potential independent variables were screened for their predictive ability based on a chi-square test (Dixon and Brown 1979):

1) Diameter (cm.) at breast height (DBH),
2) Basal area ($cm.^2$) of the tree (BA),
3) Basal area ($m.^2$) of the stand (per hectare),
4) Stand density (trees/ha.),
5) Stand age (years),
6) Site index (m. at 50 years),
7) Relative DBH,
8) Relative BA.

where: Relative BA or relative DBH for a specific tree is equal to tree BA or DBH divided by the average BA or DBH of the stand.

The following variables were selected for inclusion into the mortality function (based on significance of t-statistic at $\alpha = 0.05$). and their coefficients were estimated as :

Variable	Coefficient	S.E.	Coeff./S.E.
#trees/ha.	0.226	0.049	4.641
Rel. DBH	-2.550	0.341	-7.470
BA/ha.	0.034	0.031	2.636
Constant	3.202	0.513	-6.235

Where:

S.E. = standard error of the regression coefficient.

Coeff./S.E. = value of the t-statistic - a test of the significance of each coefficient.

An implicit assumption in the development of this function was that of uniform mortality over the time interval between measurements of the permanent sample plots. In order to accomodate this assumption and because the interval between measurments was different for different plots and for the same plot at different measurements, the dependent variable was weighted by the inverse of the number of years between measurements. In this way, the function was developed to predict the probability of annual mortality based on the assumption of uniform mortality over the measurement period. This was the approach adopted by Hamilton and Edwards (1976).

This function was then used to simulate mortality in a deterministic fashion for some of the permanent sample plot

data from which it was developed. The mortality predictions were poor for some age classes; mortality was underestimated for young stands and overestimated for old stands. Only in the mid-range of the stands (approximately 50 years of age) were the predictions reasonable. This led to two conclusions. The first was that the assumption of uniform mortality between the time of measurement of the permanent sample plots was incorrect, at least for young, dense aspen stands. This conclusion was reinforced by examination of the mortality pattern of some of the permanent sample plots. For example, the density of one plot dropped from 7580 trees per hectare at 24 years of age to 5062 trees per hectare at 29 years of age to 3802 trees per hectare at 34 years of age. Over the first five year period the mortality rate was 8.43% while over the last five year period the mortality rate was 5.9% (when compounding is taken into consideration). This illustrates that a considerable change in mortality rate can occur over a ten year span. This was the period for all of the C.F.S. permanent sample plots between the time of plot establishment and first re-measurement. It is also likely that the mortality rate changed over the five year periods as well.

Hamilton and Edwards (1976), however, obtained satisfactory mortality predictions using functions which had been developed from data assumed to have uniform mortality over the period between measurements of the plots. Hamilton and Edwards developed mortality functions for species in

northern Idaho such as western hemlock (*Tsuga heterophylla*), ponderosa pine (*Pinus ponderosa*) and subalpine fir (*Abies lasiocarpa*). These species exhibit much lower rates of mortality than trembling aspen and, therefore, have less chance of significant change in mortality rates between re-measurements of permanent sample plots. For example, the average mortality rate for western hemlock was found to be 1.36% per year. Similarly, the average mortality rate for ponderosa pine was found to be 0.95% per year. Subalpine fir had the highest reported mortality rate (2.57% per year). In contrast, the aspen stand which dropped from 7580 trees per hectare to 5062 trees per hectare over a five year period exhibited a mortality rate of 8.43% per year when compounding is taken into consideration. The second conclusion was that one function based on the logistic model was not adequate for simulating mortality in aspen, primarily because of the extremely high and variable mortality rates in young, dense stands.

To overcome these difficulties, the data were segregated into "old" stands (greater than 40 years) and "young" stands (less than 40 years). In addition, specification of model input for BMDP was revised. Tree status, the dependent variable, (0=live, 1=dead) was viewed as the "failure" or "success" of a tree in dying. If the tree "failed" to die it was assigned zero (0); if it "succeeded" in dying, it was assigned one (1). The number of years between measurements of the permanent sample plots

was viewed as the number of "chances" the tree had to die. For example, if the period between measurements was five years and the tree lived, it was assigned 0 and was considered to have failed at five opportunities to die. If, on the other hand, the tree died, it was assigned 1 but the number of opportunities to die which it actually had was unknown, since the exact year of death was unknown. If the assumption of uniform mortality between measurements was true, then the average number of opportunities for the dead trees to die would be 0.5 x the length of the measurement period, or 2.5 years in this case. If, however, mortality was greater at the beginning of the period (as the data suggest), the average number of "chances" the average mortality tree had in which to die would be less than 0.5 x the length of the period. In effect, the number of chances that the average mortality tree had in which to die was proportionally reduced. The factor used to estimate the number of annual opportunities that a dead tree actually had, on average, to die (0.50 in this case) will be referred to throughout the remainder of this thesis as the "proportional reduction factor".

Numerous mortality functions were then developed for the two groups of trees, assuming different proportional reduction factors to adjust the length of the period between measurements for the dead trees. The mortality predictions of these functions were then tested against the data from which they were developed. All of the functions from the

segregated data were better predictors than the first function which was developed from unsegregated data. In addition, the functions with a proportional reduction factor less than 0.5 were the best predictors. Problems similar to those encountered with the first mortality function occurred with all of the mortality functions developed using the data from young stands. In general, mortality in the very young stands was underestimated, while the mortality in stands approaching 40 years of age was overestimated. If the proportional reduction factor was reduced, mortality predictions for the young stands became better, but the overestimates of mortality for the older stands became worse, and vice versa when the proportional reduction factor was increased to a level near 0.5.

To overcome this difficulty, the permanent sample plot observations were further segregated into three groups on the basis of density. The three groups were:

1) Greater than 6000 trees per hectare
2) Between 2500 and 6000 trees per hectare
3) Less than 2500 trees per hectare

The logistic regression option of BMDP was used to screen the same set of potential independent variables for their predictive ability. Once again, numerous runs of BMDP were made using various values for the proportional reduction factor. The following functions appeared to outperform all others in their density class when predicting mortality for the data from which they were developed:

Function 1: Developed from 789 tree observations taken from the C.F.S. permanent sample plots with greater than 6000 stems per hectare. Proportional reduction factor was (0.30).

Variable	Coefficient	S.E.	Coeff./S.E.
Relative BA	-3.607	0.191	-18.932
Constant	0.360	0.125	2.883

Function 2:Developed from 3195 tree observations taken from the CFS permanent sample plots (from Manitoba and Saskatchewan) with 2500 to 6000 stems per hectare. Proportional reduction factor was (0.30).

Variable	Coefficient	S.E.	Coeff./S.E.
Relative BA	-3.202	0.096	-33.356
Constant	-0.256	0.058	-4.395

Function 3: Developed from 2330 tree observations taken from the Alberta Forest Service permanent sample plots with less than 2500 stems per hectare. Proportional reduction factor was (0.50).

Variable	Coefficient	S.E.	Coeff./S.E.
Age	-0.014788	.002699	-5.478
BA per ha.	0.026375	.005093	5.179
Relative BA	-1.450101	.322271	-4.500
Relative DBH	-0.913845	.512428	-1.783
BA (tree)	0.003935	.000630	6.243
DBH (tree)	-0.073380	.024800	-2.959
Constant	-1.057000	.255000	-4.139

Testing was conducted using a chi-square test (which makes use of the nominal properties of the data) and the Kolmogorov-Smirnov test (which uses the ordinal nature of the data). Function 1 was tested against two subsets of the data from which it was developed since an independent data set with greater than 6000 trees per hectare was not available (Tables 1a,1b,2a,2b, Appendix II). Function 2 was tested against an independent data set from the University of Alberta permanent sample plots in aspen (Tables 3,5, Appendix II). Function 3 was tested against an independent data set from the Alberta Forest Service permanent sample plots in aspen types (Tables 4,6, Appendix II). For all three models, using both the Chi-square and Kolmogorov-Smirnov tests, the hypothesis that each mortality function produced the same distribution of mortality as that found in real stands could not be rejected (α=0.05)

Consideration was also given to the probability predictions at the density boundary between one function and the next. On an individual tree basis, the probability of mortality would likely be disjoint at the boundary between functions, but on a stand basis, there should be no distinct deviation in mortality away from the overall trend established by the previous function. In other words, when the three mortality functions are combined in a simulation of stand mortality, there should be no distinct change in the stand mortality trend when one function is replaced by another as stand density crosses the boundary level for function selection. These three functions appeared to respond in this fashion extremely well when combined for trial simulations (see '**CONCLUSION AND RECOMMENDATIONS**').

4.2 Diameter Growth

Diameter growth, and subsequently, volume and biomass growth, was approached on an individual tree basis to ensure maximum flexibility for model development and operation. The non-linear regression package of BMDP was used for preliminary testing of the following non-linear functional forms for use in the diameter growth component of the aspen growth model:

1) Natural growth function.
2) Logistic function.
3) Allometric function.
4) Generalized Gompertz equation.

5) Modified Chapman function.

Considerable difficulties were encountered using BMDP in the estimation of coefficients for even simple forms of the above functions (eg. DBH = f(age)). In addition, it was felt that a number of independent variables might have to be included in the diameter growth function in order to reflect differences in site quality, stand density, and each tree's competitive position in the stand. These reasons combined with time limitations, forced the abandonment of non-linear forms for the diameter growth component of the aspen growth model.

Two methods of modelling diameter growth using multiple linear regression techniques were also investigated. The first method was to use diameter or basal area increment as the dependent variable. Independent variables (before transformations) were then selected from the following:

1) Present DBH or basal area (BA)
2) Number of trees per hectare
3) Stand age
4) Site index
5) Basal area (BA) per hectare
6) Relative DBH or BA (same definition as for mortality)

Variables such as relative DBH , relative BA, number of trees per hectare and BA per hectare were included as potential independent variables so that a measure of competition could be directly incorporated into the diameter

growth function.

Preliminary analysis using a random set of 205 observations selected from the C.F.S. and A.F.S. permanent sample plots indicated difficulties with this approach. Direct correlations between the independent variables and the dependent variable (both BA and DBH growth) were poor. This appeared to be due to large fluctuations in growth relative to each other, yet these fluctuations were small when compared to the diameter of the tree. For instance, two different 10 cm. trees could have their periodic diameter increment recorded as 6 mm. and 4 mm. respectively. This is a 33.33% difference when the increments are compared directly to each other as a percentage of the increment of the larger tree (i.e.(6-4)/6=.3333), but represents only a 1.89% difference when expressed as a percentage of the diameter of the larger tree (i.e.(10.6-10.4)/10.6=.0189). Lack of precision of diameter growth measurements appeared to add to this problem. In addition to general measurement errors, most of the diameter measurements from the permanent sample plots were recorded to 0.1 inches (2.54 mm.). As shown above, a difference of this magnitude between diameter measurements of two trees can be significant when compared to diameter increment.

The two alternate function forms, with significant independent variables (α=0.05) were:

1. BA increment ($cm.^2$) = 0.2029 + 5.637(Relative BA)

r^2 = 0.09878

R.S.E. = 1.99071

2. DBH increment (cm.) = -0.0273 + 0.3597(Relative DBH)
+ 0.0091(DBH)

r^2 = 0.09708

R.S.E. = 1.50600

Where: R.S.E. = relative standard error of the residuals, i.e., standard error of the residuals divided by the mean of the dependent variable

The second approach was to predict future tree diameter (rather than diameter increment) directly as a function of the same set of possible independent variables. This approach indirectly incorporates diameter increment while masking the large fluctuations of diameter increment relative to each other. An attempt was made to predict DBH (or BA) five years in the future and ten years in the future based on the same set of independent variables listed previously.

Using the same randomly selected set of 205 observations four equations were developed using multiple linear regression techniques. The dependent variable in the four equations was:

1) DBH5: DBH five years in the future

2) BA5: BA five years in the future

3) DBH10: DBH ten years in the future

4) BA10: BA ten years in the future

The four alternate functions developed, with significant independent variables (α=0.05) were:

1. DBH5(cm.) = 1.247945 + 0.9658520(DBH in cm.)
+1.663212(Relative DBH)
-0.0001481556(#trees/ha.)
-0.01882325(Age)

r^2 = 0.96273

R.S.E. = 0.10923

2. BA5($cm.^2$) =57.20455 + 1.042859(BA in $cm.^2$)
+19.59457(Relative BA)
- 0.004921721(#trees/ha.)
- 0.8118688(Age)

r^2 = 0.94367

R.S.E. = 0.25745

3. DBH10(cm.) = 2.49588 + 0.9317044(DBH in cm.)
+3.326424(Relative DBH)
-0.0002963113(#trees/ha.)

-0.03764651(Age)

r^2 = 0.86791

R.S.E. = 0.20365

4. BA10(cm.^{2}) =114.4091 + 1.085717(BA in cm.^{2})
+39.18916(Relative BA)
-0.00984344(#trees/ha.)
-1.623737(Age)

r^2 = 0.82447

R.S.E. = 0.45593

The functions predicting future DBH exhibited more favorable regression statistics than those predicting future BA. The r^2 values were much higher, and the relative standard error of residuals (defined as the standard error of the residuals divided by the mean of the dependent variable) values were lower. In addition, the residuals were more uniform over the range of predicted dependent variables. Similarly, the functions based on a five year projection were more favorable than those based on a ten year projection.

During the final phase of development of the diameter growth component it was felt that special efforts should be made to incorporate independent variables into the model which would allow maximum flexibility for model application.

Specifically, the following variables were considered to be particularly important:

1. DBH (present)

   This was the most significant independent variable in both of the preliminary regressions which had incorporated it. Intuitively, one would expect present DBH to be the best single predictor of future DBH.

2. Competition factors

   Competition may be viewed from a stand perspective (such as number of trees per hectare or basal area per hectare) or from an individual tree perspective (such as relative DBH or absolute DBH). In addition, response to thinning could potentially be viewed as a response to stand density or basal area per hectare.

3. Other Variables

   Inclusion of site index to differentiate DBH growth on sites of different quality was also thought to be important. In the preliminary regressions, however, site index was not a significant variable (α=0.05).

In addition, it was thought that a zero-intercept function might perform better than a non-zero-intercept function for the prediction of diameter growth. The y-intercept for the preliminary functions was large enough so that for small trees in a stand, a distorted overprediction of future diameter might result simply

because of the relative magnitude of the intercept and the tree diameter.

The analysis for diameter growth over a five year period was repeated with an enlarged data set consisting of 774 observations taken from the temporary sample plot data collected under this project and the Alberta Forest Service permanent sample plots.

The range of ages for this data set was 10 years to 98 years with a mean of 45.7 years. The following function was developed:

$$\begin{aligned} DBH5(cm.) = {} & 0.95890(DBH \text{ in cm}) \\ & -0.000036548x(\#tree/ha) \\ & -0.01170(AGE) + 0.074408(SITE\ INDEX) \\ & +0.036334(Relative\ DBH) \\ & +0.005842(BA \text{ in } cm.^2) \end{aligned}$$

$r^2 = 0.99684$

$R.S.E. = 0.06607$

This function was intuitively consistent (i.e. the coefficients had the theoretically "correct" signs) and incorporated variables for competition determination (Relative DBH, #trees/ha.), site quality (site index) and had a zero intercept. The #trees/ha. variable, however, was not significant ($\alpha=0.05$).

Reformulating the above function with a squared

transformation of #trees/ha. resulted in the following function (variables significant at $\alpha=0.05$):

$$\begin{aligned} \text{DBH5(cm.)} = {} & 0.9683(\text{DBH in cm}) \\ & -0.0000000027122((\#\text{tree/ha})^2) \\ & -0.0099465(\text{AGE}) \\ & +0.058651(\text{SITE INDEX}) \\ & +0.77562(\text{RELATIVE DBH}) \\ & +9.5169\text{E-}04(\text{BA in cm.}^2) \end{aligned}$$

$r^2 = 0.99849$

R.S.E. = 0.04585

This function was subsequently chosen for use in the aspen growth model because it had the highest r^2 value, the lowest R.S.E. value and a relatively uniform distribution of residuals over the range of predicted values (Figures 1,2, Appendix III). In addition, when this function was combined with the mortality functions for simulation, the subsequent pattern of diameter growth and mortality was consistent with the basic theory of tree and stand development.

4.3 Height Estimation and Site Differentiation

Height estimation and site differentiation are incorporated into the aspen growth model through the use of a site index equation. Site index, the average height of dominant and co-dominant trees at some reference age, is a

widely used and commonly accepted measure of site (Husch, Miller and Beers 1972). The regression equation was developed by J.D. Heidt, Department of Forest Science (Dancik et. al. 1980):

$$Height(m.) = 0.011292 + 0.440716(Age) - 0.002036(Age^2)$$

Site is incorporated through the use of anamorphic site index curves. The height of a tree is predicted by multiplying the height (predicted from the above equation) by the ratio of the site index for that tree to the reference site index. This function performs well up to approximately 100 years but, because it has a negative quadratic term, it becomes unrealistic after age 100. There appears to be several ways to overcome this problem. One method would be to develop a better function, possibly with a non-linear form, to prevent trees from 'shrinking' in height at very old ages. A segmented model, with a second equation for ages greater than 100, could also be used to overcome this problem. Finally, an arbitrary adjustment procedure could be incorporated into the model to allow stand height to reach a plateau, but prevent a decline. Implementation of any of these options was viewed as unnecessary for this thesis because the mean annual increment and overall maxima for biomass and volume occur before age 100. Growth projections for trembling aspen beyond 100 years of age are unreasonable for all practical

management purposes.

When this equation is used in the aspen growth model, the predicted height is assumed to be the average height of the stand. Aspen is a light intolerant species, exhibiting extremely high mortality due to suppression (Bella 1975, Bella and DeFrancheschi 1972), and thus, it has a relatively uniform overstory when compared to other species such as spruce. The height measurements from the permanent sample plots used for this project support this conclusion.

4.4 Volume

An aspen volume equation developed by Honer (1967) was used for the calculation of total volume per tree (stump and top included):

$$\text{Volume}(ft^3) = (\text{DBHOB}^2(\text{in.}))/(-0.312 + (436.683/\text{HT}(\text{ft.})))$$

This particular volume equation was used due to its' general acceptance and use across Canada. Any volume equation could, in fact, be used. The volume equation used should be selected based on how well it performs in predicting actual tree volumes in a given area for a particular tree species. No evaluation of this volume equation was carried out as part of this study.

4.5 Biomass

As with tree volume the development of equations to predict biomass was outside the scope of this thesis. Reliance was therefore placed on existing equations to predict biomass. Bella and DeFranceschi (1980) have developed a series of equations to predict tree biomass for various components of the tree that are reasonable in terms of both the geographic location of the sample and their predictive power. The results of Bella and DeFranceschi (1980) were used in the model for the calculation of total biomass per tree (stem wood, bark, branches and leaves). The function is :

$$\text{Total Tree Biomass(g.)} = -0.80319 + 0.936736(\ln(DBH^2(\text{cm.})HT.(\text{cm.})))$$

Alternative functions developed by Bella and DeFranceschi could easily be incorporated in the growth model to predict other components of tree biomass.

5. MODEL CONSTRUCTION

The aspen growth model is of the single tree/distance independent format. It was constructed to accept input data as a tree list of individual DBH's with accompanying stand parameters (such as site index). Each tree in the list is grown separately. Output is printed on a diameter class basis and includes stand characteristics such as total biomass per hectare, number of trees per hectare, etc. In addition, plots of biomass, stand density and biomass mean annual increment against stand age are produced.

The aspen growth model was formulated in APL (A Programming Language, Gilman and Rose 1976). All major routines are composed of different subordinate functions which are called by a main function as required. This modular composition, characteristic of programs written in APL, permits easy substitution of various functions or the addition of new options. For example, a different volume function (eg. for a specific area) could easily be incorporated into the aspen growth model simply by defining the new relationship in APL and then replacing the old function with this newly defined version. This inherent flexibility of APL was the primary reason for its selection as the computer language for the development of the aspen growth model. For similar reasons Hegyi (1974) used APL for the development of a growth model for jack-pine stands.

The aspen growth model operates in a *BATCH* mode which facilitates printing of output on the central printing

facilities of the University computer (see **Appendix IV**). Inputs consist of two vectors which serve as arguments to the model's principle function which is called *'MAIN'* :

INPUT1

1. Initial stand age (years)
2. Plot size (square meters)
3. Number of growing periods desired for the simulation (5 years each)
4. For output appearances only:
   - minimum diameter (*DMIN*)
   - width of diameter classes (*DWD*)
   - number of diameter classes (*DNUM*)
5. Site Index (base age 50 years)

INPUT2

1. List of DBH's (cm.) for all trees in the plot.

Trees are grown for the specified number of 5 year periods by the following steps which occur during each period:

1. Calculate biomass, volume and basal area for the trees. Convert to a per hectare value and assign the trees to diameter classes. Predicted stand height is based on age and site index.
2. Print the diameter class summary.
3. Check if maximum number of steps has been reached.

   No: Go to 4, or

   Yes: Print graphs of:

   -Biomass/ha. over age

-Trees/ha. over age

-Biomass mean annual increment (MAI).

then stop.

4. Generate a two dimensional matrix of random numbers.
   rows: same length as the input vector of tree diameters.
   columns: five (one for each year of the five year period).
5. Generate the annual probability of mortality for each tree.
6. Test each tree: is the probability of mortality greater than any of the five random numbers?
   Yes: remove that tree from the vector of DBH's, or
   No: retain the tree.
7. Grow the live trees based on the stand parameters at the beginning of the growth period.
8. Replace the vector of old diameters with the new diameters. Go to 1.

A complete list of functions used in the model, except for the plotting functions, is included in Appendix V. The plotting functions used to produce the graphs are standard library functions of the University of Alberta intended for the production of quick, relatively accurate graphs (Armitage 1972).

6. MODEL TESTING AND EVALUATION

In addition to evaluating individual mortality and diameter growth components, stand predictions based on the model were compared with permanent sample plot data. The data used in the comparisons came from the Canadian Forestry Service permanent sample plots from Riding Mountain National Park, Manitoba. The C.F.S. permanent sample plots were the only permanent plots in young aspen stands which had a number of repeated measurements. It was felt that these should be used for model testing, rather than the Alberta Forest Service permanent sample plots which were in old stands, because young stands exhibit the greatest variability in growth parameters and because the primary purpose of this model is that of biomass prediction. The data set used for testing was not independent of model development, since data from these plots were used to develop the mortality component of the aspen growth model. It was expected, therefore, that the mortality component of the aspen growth model would perform well when compared to the actual stands. The CFS plots were not used in the other components of the model, therefore, some level of independence in testing was achieved.

Plot characteristics and the tree list from the permanent sample plots at establishment were used as inputs to the aspen growth model. The growth of each stand was simulated from ages of 23 to 48 for plots 1 and 2, and from ages of 14 to 49 for plot 3. The actual stands were measured

at ages 23 (establishment), 33, 38, 44, and 49 years for plots 1 and 2, and at ages 14 (establishment), 24, 29, 35, and 40 years for plot 3. Six parameters of the simulation were compared to the real stands over the projection period:

1) Stand Volume
2) Stand Biomass
3) Stand Basal Area
4) Stand Density
5) Average DBH
6) Biomass Mean Annual Increment

Five simulation runs were conducted for each plot. The results were averaged so as to smooth out any anomalies created by the stochastic nature of the mortality component of the model.

The results of the average simulation runs are presented in graphical form in Appendices VI, VII, and VIII. In addition, comparisons of the six parameters are presented in Tables 1, 2 and 3.

The age for the predicted values is equal to the actual age for three of the five measurement points. The remaining two measurement points were only one year apart, enabling direct comparison without extrapolation.

As can be seen in Tables 1, 2, 3, the predicted results are reasonably close to the actual values. The average percent deviation of predicted from actual stand characteristics was 0.64% for the 72 combinations of plots, ages and stand characteristics predicted.

TABLE 1 - Comparison of Actual to Predicted Parameters for Plot 1.

	Age of Pred.	Pred. Value	Actual Age	Actual Value	% Diff.
VOLUME m^3/ha	23.00	134.72	23.00	134.72	.00
	33.00	249.48	33.00	243.56	2.43
	38.00	299.11	38.00	281.06	6.42
	43.00	338.31	44.00	342.93	-1.35
	48.00	359.99	49.00	360.76	-.21
BIOMASS tonnes/ha	23.00	87.23	23.00	87.23	.00
	33.00	149.57	33.00	146.14	2.35
	38.00	174.23	38.00	163.56	6.52
	43.00	192.29	44.00	193.26	-.50
	48.00	200.49	49.00	199.20	.65
BA m^2/ha	23.00	27.81	23.00	27.81	.00
	33.00	37.50	33.00	36.61	2.43
	38.00	39.96	38.00	37.55	6.42
	43.00	40.92	44.00	40.74	.45
	48.00	40.01	49.00	39.49	1.33
DENSITY trees/ha	23.00	5760.00	23.00	5760.00	.00
	33.00	3392.00	33.00	3659.00	-7.30
	38.00	2712.00	38.00	2695.00	.63
	43.00	2192.60	44.00	2138.00	2.55
	48.00	1784.80	49.00	1706.00	4.62
AVE.DBH cm	23.00	7.54	23.00	7.54	.00
	33.00	11.64	33.00	10.81	7.65
	38.00	13.49	38.00	12.87	4.79
	43.00	15.22	44.00	15.04	1.21
	48.00	16.70	49.00	16.63	.47
MAI tonnes/ha	23.00	3.79	23.00	3.79	.00
	33.00	4.53	33.00	4.43	2.36
	38.00	4.58	38.00	4.30	6.52
	43.00	4.47	44.00	4.39	1.82
	48.00	4.18	49.00	4.06	2.75

NOTE: % Diff = ((Predicted - Actual)/Actual) x 100%

TABLE 2 - Comparison of Actual to Predicted Parameters for Plot 2.

	Age of Pred.	Pred. Value	Actual Age	Actual Value	% Diff.
VOLUME m^3/ha	23.00	137.87	23.00	137.87	.00
	33.00	241.76	33.00	253.21	-4.52
	38.00	289.97	38.00	274.56	5.61
	43.00	332.47	44.00	343.87	-3.31
	48.00	353.28	49.00	375.23	-5.85
BIOMASS tonnes/ha	23.00	89.61	23.00	89.61	.00
	33.00	145.17	33.00	152.73	-4.95
	38.00	169.31	38.00	160.43	5.53
	43.00	189.54	44.00	195.09	-2.84
	48.00	197.29	49.00	208.62	-5.43
BA m^3/ha	23.00	28.46	23.00	28.46	.00
	33.00	36.34	33.00	38.06	-4.52
	38.00	38.74	38.00	36.68	5.61
	43.00	40.22	44.00	40.85	-1.56
	48.00	39.27	49.00	41.07	-4.39
DENSITY trees/h	23.00	6156.00	23.00	6156.00	.00
	33.00	3307.80	33.00	4079.00	-18.91
	38.00	2694.80	38.00	2756.00	-2.22
	43.00	2240.00	44.00	2361.00	-5.12
	48.00	1812.00	49.00	1953.00	-7.22
AVE.DBH cm	23.00	7.41	23.00	7.41	.00
	33.00	11.66	33.00	10.48	11.29
	38.00	13.37	38.00	12.64	5.81
	43.00	14.97	44.00	14.37	4.20
	48.00	16.46	49.00	15.90	3.53
MAI tonnes/ha	23.00	3.90	23.00	3.90	.00
	33.00	4.40	33.00	4.63	-4.95
	38.00	4.46	38.00	4.22	5.53
	43.00	4.41	44.00	4.43	-.59
	48.00	4.11	49.00	4.26	-3.47

NOTE: % Diff = ((Predicted - Actual)/Actual) x 100%

TABLE 3 - Comparison of Actual to Predicted Parameters for Plot 3.

	Age of Pred.	Pred. Value	Actual Age	Actual Value	% Diff.
VOLUME m^3/ha	14.00	29.26	14.00	29.26	.00
	24.00	82.32	24.00	89.97	-8.50
	29.00	113.64	29.00	108.41	4.83
	34.00	138.57	35.00	134.84	2.77
	39.00	168.13	40.00	167.54	.36
BIOMASS tonnes/ha	14.00	22.34	14.00	22.34	.00
	24.00	56.18	24.00	61.03	-7.95
	29.00	74.59	29.00	70.81	5.34
	34.00	88.16	35.00	84.92	3.81
	39.00	104.19	40.00	102.38	1.77
BA m^2/ha	14.00	12.61	14.00	12.61	.00
	24.00	21.61	24.00	23.62	-8.51
	29.00	25.25	29.00	24.09	4.83
	34.00	26.90	35.00	25.55	5.29
	39.00	29.16	40.00	28.47	2.41
DENSITY trees/ha	14.00	14519.00	14.00	14519.00	.00
	24.00	6770.40	24.00	7580.00	-10.68
	29.00	5244.60	29.00	5062.00	3.61
	34.00	4079.20	35.00	3802.00	7.29
	39.00	3417.20	40.00	3086.00	10.73
AVE.DBH cm	14.00	3.14	14.00	3.14	.00
	24.00	6.29	24.00	6.01	4.62
	29.00	7.76	29.00	7.53	3.08
	34.00	9.10	35.00	8.93	1.89
	39.00	10.36	40.00	10.46	-.91
MAI tonnes/ha	14.00	1.60	14.00	1.60	.00
	24.00	2.34	24.00	2.54	-7.94
	29.00	2.57	29.00	2.44	5.34
	34.00	2.59	35.00	2.43	6.88
	39.00	2.67	40.00	2.56	4.39

NOTE: % Diff = ((Predicted - Actual)/Actual) x 100%

In all cases, the percent difference (expressed as a percent of the actual value) was less than 20%. Only one comparison showed a difference greater than 15% (Plot 2, Age 33, Density : -18.91%). Three comparisons were greater than 10%:

1) Plot 2, Age 33, Average DBH : +11.29%
2) Plot 3, Age 24, Density : -10.68%
3) Plot 3, Age 39, Density : +10.73%

All remaining comparisons showed differences of less than 10%. It should be noted that density comparisons accounted for three of the four comparisons with greater than 10% difference between actual and predicted values, yet these plots were used to develop the mortality component of the aspen growth model. This lends support to the authors' conclusions that mortality is the most difficult component of aspen growth to model (see **'CONCLUSION AND RECOMMENDATIONS'**).

7. CONCLUSION AND RECOMMENDATIONS

Based on the testing conducted, it appears that the aspen growth model simulates stand development fairly well within the range of data from which it was developed, but caution should be taken when applying the model to situations beyond this range. As was shown in Tables 1, 2, and 3, most deviations between predicted and actual stand characteristics were less than 10%, with an average percentage deviation of 0.64%. Additional testing, with fully independent data from the prairie provinces, is necessary to verify this conclusion. However, no such data are available for the geographic region under consideration.

In order to improve the aspen growth model, additional research into the pattern and prediction of mortality in aspen is necessary. Mortality was the most difficult component of aspen growth to model due to its extremely high variability in young aspen stands. High levels of mortality can have drastic effects on other growth parameters such as stand biomass or volume. One problem in particular hampered efforts to model mortality in aspen stands. This was the length of time between measurements of the permanent sample plots, particularly in young stands. In most cases the time period between measurements in the CFS permanent sample plots was five years, but in some it was ten years. Aspen stands between ten and thirty years of age exhibit such high and variable levels of mortality that their density may be halved in ten years, with substantial changes in mortality

rate over that period. This presents a problem when estimating coefficients for the logistic function to predict the probability of individual tree mortality. In particular, when specifying the model for a logistic regression package, the average time of mortality must be estimated for the trees which died between measurements. When the stand is in a state of high and variable mortality, specifying this average time of mortality becomes both crucial, and extremely difficult. Annual re-measurements, particularly in young stands, are essential for monitoring mortality in aspen.

Segregating aspen stands into three groups on the basis of stand density (see '**BASIC MODEL RELATIONSHIPS - Mortality**') enabled satisfactory mortality predictions over each corresponding density range but created problems due to the need to link these functions in the aspen growth model. A segmented approach was adopted to join these three functions but this created the possibility of disjoint density trends when the model is used for simulation runs which utilize two or more of the mortality functions. While this problem was not evident in the simulation runs conducted for this study, more research into the linkage of such functions for growth modelling is needed. One potential approach is the use of spline functions (DeBoor 1978, Greville 1968), but this is beyond the scope of this thesis.

Consideration of other aspects of the aspen growth model also lead to favorable conclusions. Modular component

structure in APL allows easy substitution of various functions or the incorporation of new options. The aspen growth model produces output both in graphical and tabular form, allowing easy interpretation of results for management purposes.

More research is necessary if new options are to be incorporated into the aspen growth model. Such options might include:

1) Clonal variation,
2) Harvesting,
3) Fire,
4) Insects,
5) Disease.

As previously stated, the incorporation of any of the above-mentioned options is facilitated by the structure of the aspen growth model.

As it exists, the aspen growth model has a number of potential management uses. Of obvious importance is the prediction of growth and growth rates for various parameters such as biomass and volume. For such uses, the aspen growth model can replace more conventional methods of stand growth estimation, such as stand-table projection. In addition, the aspen growth model has a distinct advantage over general yield tables in that, with a tree list of diameters from a stand and the other model inputs, the volume or biomass yield of that stand can be predicted for any reasonable length of time, to any reasonable stand age. The manager

can, therefore, obtain a growth projection based on the existing stand parameters and can choose the format for the results of this projection. By altering input variables, the manager can explore growth responses. Aside from the simpler uses, the aspen growth model has potential for linkage with linear programming models such as *Timber RAM* (Hennes et al. 1971). Inventory or permanent sample plot data could serve as direct inputs to the aspen growth model. Growth projections would then serve as input to the linear programming model which would then solve for optimal production (eg. volume) for an area based on specified constraints.

These are only a few of the potential management uses for the aspen growth model. It is evident that the actual use of this model will depend, to a large degree, on the level of utilization of the aspen resource in Canada, and the subsequent need for growth and yield information of better quality than is presently available.

8. LITERATURE CITED

Alemdag, I.S. and T.G. Horton. 1972. *Is decay volume strongly related to number of knots and log volume in poplar?* Canadian Department of the Environment, Bi-Monthly Research Notes 28(2-3):18.

Anderson, G.W. 1958. *Hypoxylon infection not greatly influenced by sex of aspen trees.* U.S.D.A., Forest Service, Lake States Forest Experiment Station, Technical note 535, 1P.

Anderson, G.W., and R.L. Anderson. 1968. Relationship between density of quaking aspen and incidence of Hypoxylon canker. *Forest Science* 14(1): pp.107-112.

Anderson, R.L. 1952. Factors influencing the incidence of Hypoxylon canker of aspen. *Phytopathology* 42: p. 463.

Anderson, R.L. 1964. Hypoxylon canker impact on aspen. *Phytopathology* 54: 253-257.

Anderson, R.L. and A.L.Schipper,Jr. 1978. A system for predicting the amount of *Phellinus (Fomes) igniarius* rot in trembling aspen stands. U.S.D.A. Forest Service, North Central Forest Experiment Station, Research Note NC-232. 4p.

Andrejak, G.E. and B.V. Barnes. 1969. A seedling population of aspens in southeastern Michigan. *The Michigan Botanist* 8:189-202.

Armitage, E.G. 1972. *APL Public Library Documentation.* Computing Services Reference Manual, University of Alberta, Edmonton, Alberta.

Bailey, G.R. and J. Dobie. 1977. *Alberta poplars--tree and log quality.* Department of Fisheries and the Environment, Forestry Directorate. Western Forest Products Laboratory Information Report VP-X-155, 8p.

Baranyay, J.A. 1967. Notes on Hypoxylon canker of aspen in Alberta. *Forestry Chronicle:* 373-380.

Barley, A.W. and R.A.Wroe. 1974. Aspen invasion in a portion of the Alberta parklands. *Journal of Range Management* 27:263-266.

Barnes, B.V. 1966. The clonal growth habit of American aspens. *Ecology* 47(3):439-447.

Barnes, B.V. 1969. Natural variation and delineation of clones of *Populus tremuloides* and *P. grandidentata* in

northern lower Michigan. *Silvae Genetica* 18:130-142.

Barnes, B.V. 1975. Phenotypic variation of Trembling Aspen in western North America. *Forest Science* 21(3):319-328.

Basham, J.T. 1958. Decay of trembling aspen. *Canadian Journal of Botany* 36: 491-505.

Basham, J.T. 1960. The effects of decay on the production of trembling aspen pulpwood in the Upper Pic region of Ontario. Canadian Department of Agriculture. Forestry Biology Division Studies in *Forest Pathology XXI*, 25p.

Bella, I.E. 1970. *Simulation of growth, yield and management of aspen.* Ph.D. Thesis, University of British Columbia. 190 p.

Bella, I.E. 1975. *Growth-density relations in young aspen sucker stands.* Environment Canada, Canadian Forestry Service, Northern Forestry Research Centre, Edmonton, Alberta, Information Report NOR-X-124. 12p.

Bella, I.E. and J.P. DeFranceschi. 1972. *The effect of logging practices on the development of new aspen stands, Hudson Bay, Saskatchewan.* Northern Forest Research Centre, Information Report NOR-X-33. 20 p.

Bella, I.E. and J.P. DeFranceschi. 1980. *Biomass productivity of young aspen stands in western Canada.* Environ. Can., Can. For. Serv., North. For. Res. Cent. Edmonton,Alberta. Inf. Rep. NOR-X-219.

Berry, A.B. and W.M. Stiell. 1978. Effect of rotation length on productivity of aspen sucker stands. *Forestry Chronicle* 54:265-7.

Bier, J.E. 1940. *Studies in forest pathology. III. Hypoxylon canker of poplar.* Canadian Department of Agriculture Technical Bulletin 27, 40p.

Brinkman, K.A. and E.I.Roe. 1975. *Quaking aspen : Silvics and management in the Lake States.* U.S.D.A. Forest Service, Agricultural Handbook no. 486. 52p.

Buckman, R.E. and L.H. Blankenship. 1965. Repeated spring prescribed burning reduced abundance and vigor or aspen root suckering. *Journal of Forestry* 65: 23-25.

Christensen, C.M., R.L. Anderson, A.C. Hodson, and P.O. Rudolf. 1951. *Enemies of aspen.* United States Department of Agriculture, Forest Service. Lake States Forest Experimental Station, Lake States Aspen Report No. 22, 16p.

Clutter, J.L. and D.M. Belcher. 1978. *Yield of site-prepared slash pine plantations in the lower coastal plain of Georgia and Florida.* IN: Growth Models for Long Term Forcasting of Timber Yields. Fries, Burhart and Max (Eds.). Virg. Poly. Inst. and St. Univ. Sch. For. and Wild. Res. Pub. FWS-1-78.

Copony, J.A. and B.V. Barnes. 1974. Clonal variation in the incidence of Hypoxylon canker on trembling aspen. *Canadian Journal of Botany* 52:1475-1481.

Dancik, B.P., J.D. Heidt, K.O. Higginbotham, and S.J. Titus. 1980.*Development of a stand growth model for trembling aspen in the prairie provinces of Canada.* Final Report, Phase One, ENFOR Project No. P-102.

Davidson, A.G. and R.M. Prentice. 1968. *Insects and diseases.* In: Growth and Utilization of Poplars in Canada. Ministry of Forestry and Rural Development. Editors: J.S. Maini and J.H. Cayford. pp 116-144.

Day, M.W. and F.C. Strong. 1959. *A study of Hypoxylon canker on aspen.* Michigan Agricultural Experimental Bulletin 41:870-877.

DeByle, N.V. 1976. *The aspen forest after harvest.* In:Utilization and marketing as tools for aspen management in the Rocky Mountains, Proceedings of the Symposium, p.20-23. U.S.D.A. Forest Service General Technical Report RM-29, Rocky Mountain Forest and Range Experiment Station, Fort Collin, Colo.

DeBoor, C. 1978. *A Practical Guide to Splines.* Springer-Verlag New York Inc., 392 p.

Dixon, W.J. and M.B. Brown. 1979. *BMDP-79 Biomedical Computer Programs P-Series.* University of California Press, Berkely, 380 p.

Duncan, A.G. and A.C. Hodson. 1958. Influence of the forest tent caterpillar upon the aspen forests of Minnesota. *Forest Science*, 4(1):71-93.

Egeberg, R. 1963. Inherent variation in the response of aspen to frost damage. *Ecology* 44:153-156.

Ek, A.R. 1974. Nonlinear models for stand table projection in northern hardwood stands. *Can. J. For. Res.*4:23-27.

Ek, A.R. and A. Dudek. 1980. *Development of tree based stand growth simulators: progress and applications.* Staff Paper Series No. 20, Department of Forest Resources College of Forestry, University of Minnesota, St. Paul, Minn.

Farmer, R.E.,Jr. 1962. Aspen root sucker formation and apical dominance. *Forest Science* 8: 403-410.

Fralish, J.S. and O.L. Loucks. 1967. *Prediction of aspen site index from correlation with site factors in Wisconsin.* University of Wisconsin, Forestry Research Notes, No. 132.

French, J.R. and P.D. Manion. 1975. Variability of host and pathogen in Hypoxylon canker of aspen. *Canadian Journal of Botany* 53: 2740-2744.

Garrett, P.W. and R. Zahner. 1964. Clonal variation in suckering of aspen obscures effect of various clearcutting treatments. *Journal of Forestry* 62:749-750.

Gilman, L. and A.J. Rose. 1974. *APL, An Interactive Approach.* John Wiley and Sons, Inc. 378 p.

Graham, S.A. and R.P. Harrison. 1954. Insect attacks and Hypoxylon infections in aspen. *Journal of Forestry* 52:741-743.

Greville, T.N.E. 1968. *Theory and Application of Spline Functions.* Academic Press, New York, 212 p.

Gruenhagen, R.H. 1945. Hypoxylon pruinatum and its pathogenesis on poplar. *Phytopathology* 35: 72-89.

Hamilton, D.A. Jr. 1974. *Event probabilities estimated by regression.* U.S.D.A. Forest Service, Intermountain Forest and Range Experiment Station, Research Paper INT-152.

Hamilton, D.A. Jr. 1980. *Modeling mortality : a component of growth and yield modeling.* Paper presented at the Forecasting Forest Stand Dynamics Workshop, Lakehead University,Thunder Bay, Ontario.

Hamilton, D.A. Jr. and B.M. Edwards. 1976. *Modeling the probability of individual tree mortality.* U.S.D.A. Forest Service, Intermountain Forest and Range Experiment Station, Research Paper INT-185.

Hamilton, D.A. Jr. and D.L.R. Wendt. 1975. *SCREEN:a computer program to identify predictors of dichotomous dependent variables.* U.S.D.A. Forest Service, Intermountain Forest and Range Experiment Station, General Technical Report INT-22.

Hann, D.W. 1979. *Evaluation of the utility of various stand growth models in managerial decision making.* (unpublished).

Hegyi, F. 1974. *A simulation model for managing jack-pine stands*. IN: Growth models for tree and stand simulation, Fries(Ed.). Dept. Forest Yield Res., Royal Coll. For., Stockholm.

Hennes, L.C., Irving, M.J., and D.L.Navon. 1971. *Forest control and regulation...a comparison of traditional methods and alternatives.* United States Department of Agriculture, Forest Service Research Note PSW-231, 10 p.

Hinds, T.E. 1963. *Extent of decay associated with Fomes igniarius sporophores in Colorado aspen.* United States Forest Service Research Note RM 4, 4p.

Hinds, T.E. and R.G. Krebill. 1975. *Wounds and canker diseases on western aspen.* United States Department of Agriculture, Forest Service, Forest Pest Leaflet 152, 9p.

Hinds, T.E. and E.M. Wengert. 1977. *Growth and decay losses in Colorado aspen.* United States Department of Agriculture, Forest Service Research Paper Rm-193, 10p.

Honer, T.G. 1967. *Standard volume tables and merchantable conversion factors for the commercial tree species of central and eastern Canada.* Forest Management Research and Services Institute, Ottawa, Ontario, Information Report FMR-X-5.

Husch, B., C.I. Miller, and T.W. Beers. 1972. *Forest Mensuration.* The Ronald Press Company, New York, 410 p.

Ives, W.G.H. 1971. *The forest tent caterpillar in Alberta.* Northern Forest Research Centre. Internal Report NOR-4, 30p.

Jarvis, J.M., G.A. Steneker, R.M. Wadron and J.C.Lees. 1966. *Review of Silvicultural Research, White Spruce and Trembling Aspen Cover Types - Mixed Wood Forest Section, Boreal Forest Region - Alberta - Saskatchewan - Manitoba.* Ministry of Forestry and Rural Development, Ottawa. 189 p.

Jones, J.R. and D.P. Trujillo. 1975. *Height-growth comparisons of some quaking aspen clones in Arizona.* United States Department of Agriculture, Forest Service Rocky Mountain Forest and Range Experimental Station Research Note RM-282, 4p.

Kaufert, F.H. 1948. *The preservative treatment of aspen.* U.S.D.A. Forest Service, Lake States Forest Experiment Station, Lake States Aspen Report No.19. 19p.

Kemperman, J.A. 1977. *Aspen clones: Development, variability and idenfication.* Ontario Ministry of Natural Resources Division of Forests, Forest Research Branch, Forest Research Information Paper 101. 11 p.

Kemperman, J.A. 1978. *Sucker-root relationships in aspen.* Ontario Ministry of Natural Resources, Forest Research Note No. 12, 4p.

Kemperman, J.A., N.F. Lyon, and S. Navratil. 1976. *Incidence and volume of defect in second growth aspen stands in northern Ontario.* Ontario Ministry of Natural Resources, Division of Forestry, Forest Research Branch, Forest Research Report 10.

Kemperman, J.A., S. Navratil, and J.T. Basham. 1978. *Preliminary assessment of defect variation among aspen clones in northern Ontario.* Ministry of Natural Resources. Ontario Forest Research Centre, Forest Research Report No. 104, 9p.

Kirby, C.L. 1962. *Growth and yield of white spruce - aspen stands in Saskatchewan.* Saskatchewan Department of Natural Resources, Forestry Branch, Technical bulletin No. 4. 58p.

Kirby, C.L., W.S. Bailey and J.G. Gilmour. 1957. *The Growth and Yield of Aspen in Saskatchewan.* Saskatchewan Deparment of Natural Resources Technical Report No.3. 67 p.

Lehn, G. 1979. *Stem Biomass Production among Aspen clones.* Master's thesis, University of Alberta. 72 p.

Maini, J.S. 1967. Variation in the vegetative propogation of *Populus* in natural populations. *Bulletin Ecological Society of America* 48: 75-76.

Maini, J.S. and R.W.Horton. 1966. Vegetative propogation of *Populus* spp. I. Influence of temperature on formation and initial growth of aspen suckers. *Canadian Journal of Botany* 44:1183-1189.

McDonough, W.T. 1979. *Quaking aspen-seed germination and early seedling growth.* USDA Forest Service, Intermountain Forest and Range Experiment Station, Research Paper INT-234, 13 pp.

Meyer, D. 1956. *The relation of soil horizon, texture, and acidity to the site index of aspen in northern Minnesota.* University of Minnesota, School of Forestry, Minnesota Forestry Notes No. 52, 2p.

Moser, J.W. 1972. Dynamics of an uneven-aged forest stand.

For. Sci. 18(3):184-191.

Moser, J.W. and O.F. Hall. 1969. Deriving growth and yield functions for unevenaged forest stands. *For. Sci.* 15:183-188 .

Munro, D. 1974. *Forest growth models - a prognosis.* IN: Growth models for tree and stand simulation, Fries(Ed.). Dept. Forest Yield Res., Royal Coll. For., Stockholm.

Navratil, S. 1972. *Poplar. 1) Internal quality of second growth aspen stands. 2) Aspen clonal study. 3) Silvicultural quality of second growth aspen stands.* From: Annual Program Report of the Forest Resources Branch, Divison of Forestry, Ministry of Natural Resources, Ontario, January 1 to December 31, 1972, p. 64-69.

Newnham, R.M. and J.H.G. Smith. 1964. Development and testing of stand models for Douglas-fir and lodgepole pine. *For. Chron.* 40(4): 494-502.

Peden, L.M., J.S. Williams and W.E. Frayer. 1973. A Markov model for stand table projection. *For. Sci.* 19(4):303-314.

Perala, D.A. 1977. *Manager's handbook for aspen in the north-central States.* U.S.D.A. For. Serv., North Cent. For. Exp. Stn., Gen. Tech. Rep. NC-36, 30p.

Pienaar, L.V. and K.J. Turnbull. 1973. The Chapman-Richards generalization of Von Bertalanffy's growth model for basal area growth and yield in even-aged stands. *For. Sci.* 19:2-22.

Povah, A. 1924. Hypoxylon poplar canker. *Phytopathology* 14:140-145.

Riley, C.G. 1952. *Fomes igniarius* decay of poplar. *Canadian Journal of Botany* 30:710-734.

Riley, C.G. and J.E. Bier. 1936. Extent of decay in poplar as indicated by the presence of sporophores of the fungus *Fomes igniarius* Linn. *Foresty Chronicle* 12:249-253.

Sandberg, D. and A.E. Schneider. 1953. *The regeneration of aspen by suckering.* Univ. Minnesota For. Notes 24, 2p., St. Paul, MN.

Schier, G.A. 1973a. Effects of gibberelic acid and an inhibitor of gibberellin action on suckering from aspen root cuttings. *Canadian Journal of Forest Research* 3:39-44.

Schier, G.A. 1973b. Origin and development of aspen root suckers. *Canadian Journal of Forest Research* 3:45-53.

Schier, G.A. 1973c. Effect of abscisic acid on sucker development and callus formation on excised roots of *Populus tremuloides. Physiol. Plant.* 28:143-145.

Schier, G.A. 1973d. Seasonal variation in sucker production from excised roots of *Populus tremuloides* and the role of endogenous auxin. *Canadian Journal of Forest Research* 3:459-461.

Schier, G.A. 1974. Vegetative propogation of aspen: clonal variation in suckering from root cuttings and in rooting of sucker cuttings. *Canadian Journal of Forest Research* 4:465-467.

Schier, G.A. 1975. Promotion of sucker development on *Populus tremuloides* root cuttings by an antiauxin. *Canadian Journal of Forest Research* 5(2): 338-340.

Schier, G.A. 1976. *Physiological and environmental factors controlling vegetative regeneration of aspen.* In: Utilization and marketing as tools for aspen management in the Rocky Mountains, Proceedings of the Symposium, p.20-23. U.S.D.A. Forest Service General Technical Report RM-29, Rocky Mountain Forest and Range Experiment Station. Fort Collins. Colo.

Schier, G.A. and R.S. Johnston. 1971. Clonal variation in total non structural carbohydrates of trembling aspen roots in three Utah areas. *Canadian Journal of Forest Research* 1:252-255.

Schier, G.A. and A.D. Smith. 1979. *Sucker regeneration in a Utah aspen clone after clear cutting, partial cutting, scarification, and girdling.* United States Department of Agriculture, Forest Service, Intermountain Forest and Range Experiment Station, Research Note INT-253, 6 p.

Schier,G.A. and J.C.Zasada. 1973. Role of carbohydrate reserves in *Populus tremuloides. Canadian Journal of Forest Research.* 2:243-250.

Sorensen, R.W. 1968. *Size of aspen crop trees little affected by initial sucker density.* United States Department of Agriculture, Forest Service. North Central Forest Experimental Station, Research Note NC-51, 4p.

Stage, A.R. 1973. *Prognosis model for stand development.* U.S.D.A. Forest Service Res. Pap. INT-137. 32p.

Steneker, G.A. 1966. *Thinning aspen Riding Mountain.* Forestry Research Laboratory Information Report MS-X-2,

11p.

Steneker, G.A. 1974. Factors affecting the suckering of trembling aspen. *Forestry Chronicle* 50(1): 32-34.

Steneker, G.A. 1976. *Guide to the silvicultural management of trembling aspen in the prairie provinces.* Northern Forestry Research Centre, Information Report NOR-X-164, 6p.

Steneker, G.A. and R.E.Wall. 1970. *Aspen clones - their significance and recognition.* Forest Research Laboratory. Liason and Service Note MS-L-8. 10p.

Steneker, G.A. and M.A.Walters. 1971. *The effect of root length upon the suckering of trembling aspen.* Forest Research Laboratory, Edmonton, Alberta. Information Report A-X-46. 11p.

Stoeckler, J.H. 1948. The growth of quaking aspen as affected by soil properties and fire. *Journal of Forestry.* 46: 727-737.

Stoeckler, J.H. 1960. *Soil factors affecting the growth of quaking aspen forests in the Lake States.* University of Michigan Agriculture Station, Technical Bulletin No. 233, 48p.

Stoeckler, J.H. and J.W. Macon. 1956. Regeneration of aspen cutover areas in northern Wisconsin. *J. For.* 54:13-16.

Strothmann, R.O. 1960. *Evaluating the growth potential of aspen lands in northern Minnesota.* United States Department of Argiculture, Forest Service, Lake States Forest Experiment Station, Paper No. 86, 20p.

Strothmann, R.O. and M.L. Heinselman. 1957. *Five-year results in an aspen sucker density study.* United States Department of Agriculture, Forest Service, Lake States Forest Experimental Station, Technical Note 490, 2p.

Thomas, G.P., D.E. Etheridge and G. Paul. 1960. Fungi and decay in aspen and balsam poplar in the Boreal Forest Region, Alberta. *Canadian Journal of Botany* 38:459-466.

United States Department of Agriculture, Forest Service. 1979. A generalized forest growth projection system applied to the lake states region. U.S.D.A. Gen. Tech. Rep. NC-49. 96p.

Wall, R.E. 1969. *Distribution of Fomes igniarius in aspen stands as affected by clonal variation.* Canadian Deparment of Fisheries and Forestry, Bi-monthly Research Notes, 25(1):5.

Wall, R.E. 1971. Variation in decay in aspen stands as affected by their clonal growth pattern. *Canadian Journal of Forest Research* 1:141-146.

Zahner, R. and N.A. Crawford. 1965. *The clonal concept in aspen site relations*, p.229-243. In C.T. Youngbery (ed.). Forest-soil Relationships in North America. Oregon State University Press, Corvallis, Oregon.

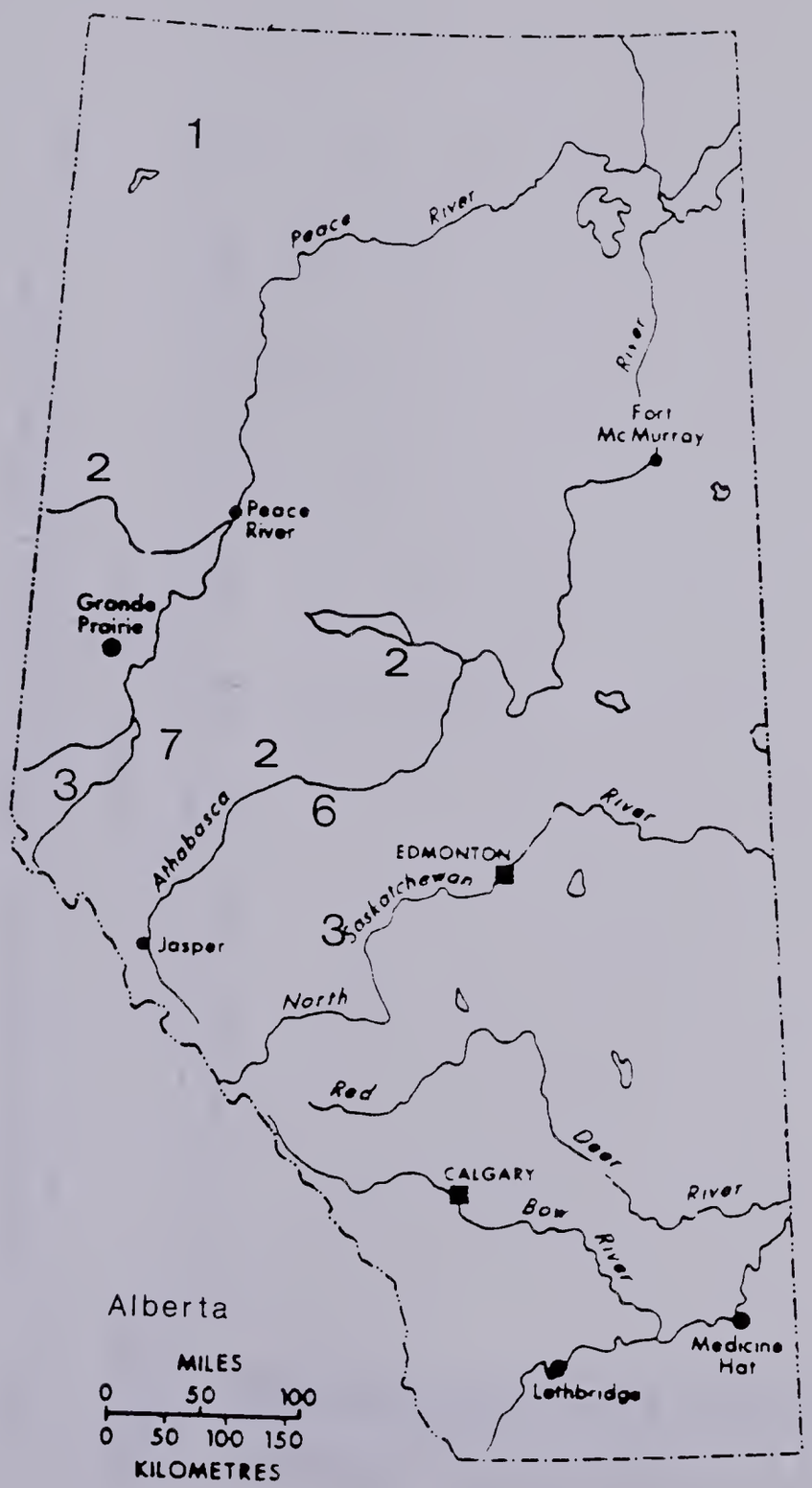
Zasada, J.C. and G.A. Schier. 1973. Aspen root suckering in Alaska: effect of clone, collection data, and temperature. *Northwest Science* 47(2): 100-104.

Zilgitt, W.M. 1951. *Disking to increase stocking in aspen stands.* U.S.D.A. Forest Service, Lake States Forest Experiment Station Technical Note 357:1p.

APPENDIX I - Locations and AGE/DENSITY Summaries for A.F.S. Permanent Sample Plots and ENFOR Temporary Sample Plots.

Figure1 Alberta Forest Service

Permanent Sample Plots

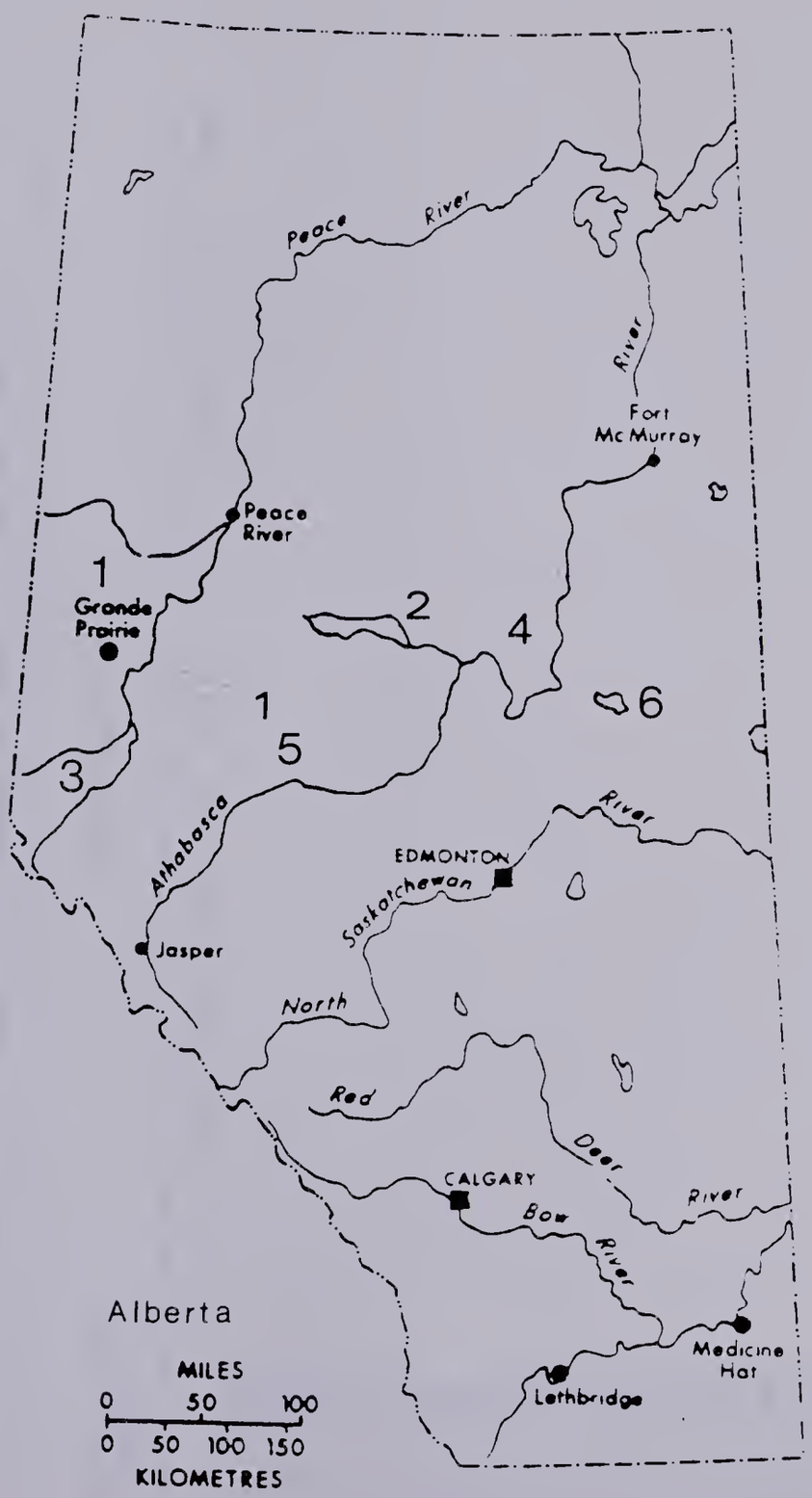


Note: Numbers on map represent the number of plots located in that area.

Table 1: DISTRIBUTION OF ASPEN PSP'S (ALBERTA) BY DENSITY (STEMS/HA) AND AGE

Density Class (Stems/ha.)	Age Class (Years)										
	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100	>100
0- 250											
251- 500						3			4	7	
501- 750						10	1		4	1	
751-1000						7	2	3	5		
1001-1250						9		3	6		2
1251-1500						6	1	2	3		
1501-1750						2			1		
1751-2000						3			1		1
2001-2250						5					1
2251-2500						2					
2501-2750						1					
2751-3000											
3001-3250											
3251-3500											
3501-3750											
3751-4000											
4001-4250											
4251-4500											
4501-4750											
4751-5000											
5001-5250											
5251-5500											
5501-5750											
5751-6000											
6001-6250											
>6251											

Figure 2 ENFOR Temporary Sample Plots



Note: Numbers on map represent the number of plots located in that area.

Table 2: DISTRIBUTION OF ASPEN TSP'S (ENFOR) BY DENSITY (STEMS/HA) AND AGE

Density Class (Stems/ha.)	Age Class (Years)										
	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100	>100
0- 250											
251- 500											
501- 750											
751-1000											
1001-1250											
1251-1500											
1501-1750						1					
1751-2000					1						
2001-2250				1		1					
2251-2500					1						
2501-2750					2	1					
2751-3000											
3001-3250					1						
3251-3500					1	1					
3501-3750											
3751-4000				1							
4001-4250				1							
4251-4500				2							
4501-4750			1								
4751-5000											
5001-5250				1							
5251-5500				1							
5501-5750											
5751-6000			1	1							
6001-6250				1							
>6251		2									

APPENDIX II - Chi-square Tests and Kolmogorov-Smirnov Tests for Mortality Functions.

Table 1a Chi-Square Tests

Test 1. Mortality Function #1 (for > 6000 trees/ha.)

DBH CLASS	PERCENT	TALLY	OBSERVED	EXPECTED	CHI-STAT
0 - 1	0.340	2.000	2.000	2.000	0.000
1 - 2	15.476	91.000	91.000	90.067	0.000
2 - 3	26.701	157.000	139.000	130.634	0.007
3 <	57.483	338.000	49.000	60.523	2.194
TOTALS	100.000	588.000	281.000	283.223	2.736

Critical value of CHI-square @ 0.05 level with 3 d.f.= 7.81

Table 1b Chi-Square Tests

Test 2. Mortality Function #1 (for > 6000 trees/ha.)

DBH CLASS	PERCENT	TALLY	OBSERVED	EXPECTED	CHI-STAT
2 - 3	0.977	3.000	3.000	2.829	0.010
3 - 4	14.332	44.000	42.000	37.126	0.640
4 - 5	18.241	56.000	32.000	36.033	0.451
5 - 6	17.264	53.000	15.000	19.803	1.165
6 <	49.186	151.000	10.000	11.884	0.299
TOTALS	100.000	307.000	102.000	107.675	2.565

Critical value of CHI-square @ 0.05 level with 4 d.f.= 9.49

Table 2a Kolmogorov-Smirnov Tests

Test 1. Mortality Function #1 (for > 6000 trees/ha.)

DBHCLASS (CM.)	OBS RELFREQ	O OBS CUMFREQ	PRED RELFREQ	P PRED CUMFREQ	|P - O|
0 - 1	.00712	.00712	.00706	.00706	.00006
1 - 2	.32384	.33096	.31801	.32507	.00589
2 - 3	.49466	.82562	.46124	.78631	.03932
3 - 4	.14235	.96797	.19834	.98465	.01667
4 - 5	.03203	1.00000	.01522	.99987	.00013
5 - 6	.00000	1.00000	.00013	1.00000	.00000

MAXIMUM |P - O| IS : .03932

Sample size = 281 (actual mortality)
Critical K-S statistic @ 0.05 = .0811

Table 2b Kolmogorov-Smirnov Tests

Test 2. Mortality Function #1 (for > 6000 trees/ha.)

DBHCLASS (CM.)	OBS RELFREQ	O OBS CUMFREQ	PRED RELFREQ	P PRED CUMFREQ	|P - O|
2 - 3	.02941	.02941	.02627	.02627	.00314
3 - 4	.41176	.44118	.34480	.37107	.07011
4 - 5	.31373	.75490	.33465	.70572	.04919
5 - 6	.14706	.90196	.18391	.88963	.01233
6 - 7	.05882	.96078	.08942	.97906	.01827
7 - 8	.00980	.97059	.01837	.99743	.02684
8 - 9	.00000	.97059	.00227	.99969	.02911
9 - 10	.01961	.99020	.00029	.99998	.00979
10 - 11	.00980	1.00000	.00002	1.00000	.00000

MAXIMUM |P - O| IS : .07011

Sample size = 102 (actual mortality)
Critical K-S statistic @ 0.05 = .1347

Table 3 Chi-Square Tests

Test 1. Mortality Function #2 (for 2500< < 6000 trees/ha.)

DBH CLASS	PERCENT	TALLY	OBSERVED	EXPECTED	CHI-STAT
1 - 4	3.562	13.000	10.000	9.606	.016
4 - 5	7.123	26.000	14.000	16.966	.519
5 - 6	11.233	41.000	23.000	23.377	.006
6 - 7	13.425	49.000	17.000	22.718	1.439
7 - 8	10.137	37.000	13.000	12.828	.002
8 - 9	11.781	43.000	9.000	10.491	.212
9 - 10	5.479	20.000	6.000	3.188	2.481
10 <	37.260	136.000	6.000	4.213	.758
TOTALS	100.000	365.000	98.000	103.387	5.433

Critical value of CHI-square @ 0.05 level with 7 d.f.= 14.07

Table 4 Chi-Square Tests

Test 1. Mortality Function #3 (for < 2500 trees/ha.)

DBH CLASS	PERCENT	TALLY	OBSERVED	EXPECTED	CHI-STAT
< 18	6.383	9.000	7.000	4.103	2.045
18 - 21	8.511	12.000	3.000	4.239	.362
21 - 24	11.348	16.000	6.000	4.932	.231
24 - 27	24.823	35.000	6.000	9.463	1.267
27 - 30	18.440	26.000	3.000	6.284	1.716
30 - 33	19.149	27.000	1.000	5.896	4.066
33 <	11.348	16.000	1.000	3.131	1.450
TOTALS	100.000	141.000	27.000	38.046	11.137

Critical value of CHI-square @ 0.05 level with 6 d.f.= 12.50

Table 5 Kolmogorov-Smirnov Test

Test 1. Mortality Function #2 (for 2500 < < 6000 trees/ha.)

DBHCLASS (CM.)	OBS RELFREQ	O OBS CUMFREQ	PRED RELFREQ	P PRED CUMFREQ	|P - O|
0 - 1	.01020	.01020	.00794	.00794	.00226
3 - 4	.09184	.10204	.08497	.09291	.00913
4 - 5	.14286	.24490	.16410	.25702	.01212
5 - 6	.23469	.47959	.22611	.48313	.00354
6 - 7	.17347	.65306	.21973	.70286	.04980
7 - 8	.13265	.78571	.12408	.82694	.04123
8 - 9	.09184	.87755	.10147	.92841	.05086
9 - 10	.06122	.93878	.03083	.95925	.02047
10 - 11	.03061	.96939	.02511	.98436	.01497
11 - 12	.02041	.98980	.00985	.99421	.00441
12 - 13	.01020	1.00000	.00287	.99708	.00292
13 - 14	.00000	1.00000	.00223	.99931	.00069
14 - 15	.00000	1.00000	.00040	.99971	.00029
15 - 16	.00000	1.00000	.00015	.99986	.00014
16 - 17	.00000	1.00000	.00011	.99998	.00002
17 - 18	.00000	1.00000	.00001	.99999	.00001
18 - 19	.00000	1.00000	.00001	1.00000	.00000

MAXIMUM |P - O| IS : .05086

Sample size = 98 (actual mortality)
Critical K-S statistic @ 0.05 = .1374

Table 6 Kolmogorov-Smirnov Test

Test 1. Mortality Function #3 (for < 2500 trees/ha.)

DBHCLASS (CM.)	OBS RELFREQ	O OBS CUMFREQ	PRED RELFREQ	P PRED CUMFREQ	|P - O|
9 - 12	.00000	.00000	.01597	.01597	.01597
12 - 15	.11111	.11111	.03882	.05479	.05632
15 - 18	.14815	.25926	.05303	.10782	.15144
18 - 21	.11111	.37037	.11142	.21923	.15114
21 - 24	.22222	.59259	.12963	.34887	.24373
24 - 27	.22222	.81481	.24871	.59758	.21724
27 - 30	.11111	.92593	.16517	.76275	.16318
30 - 33	.03704	.96296	.15497	.91772	.04525
33 - 36	.03704	1.00000	.06286	.98058	.01942
36 - 39	.00000	1.00000	.01474	.99532	.00468
39 - 42	.00000	1.00000	.00468	1.00000	.00000

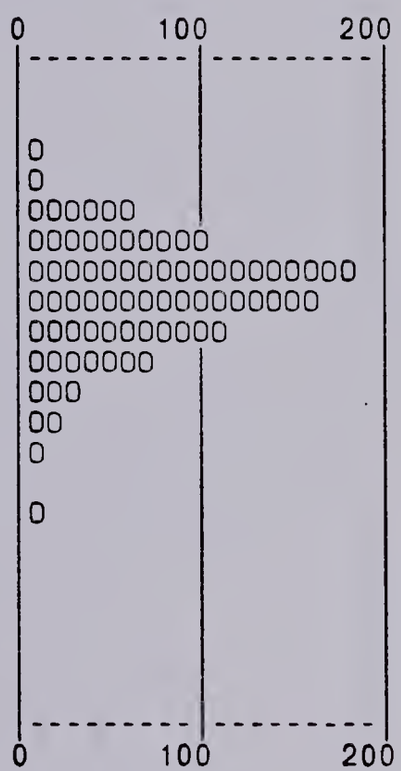
MAXIMUM |P - O| IS : .24373

Sample size = 27 (actual mortality)
Critical K-S statistic @ 0.05 = .2617

APPENDIX III - Histogram and Plot of Residuals for Final Diameter Growth Function.

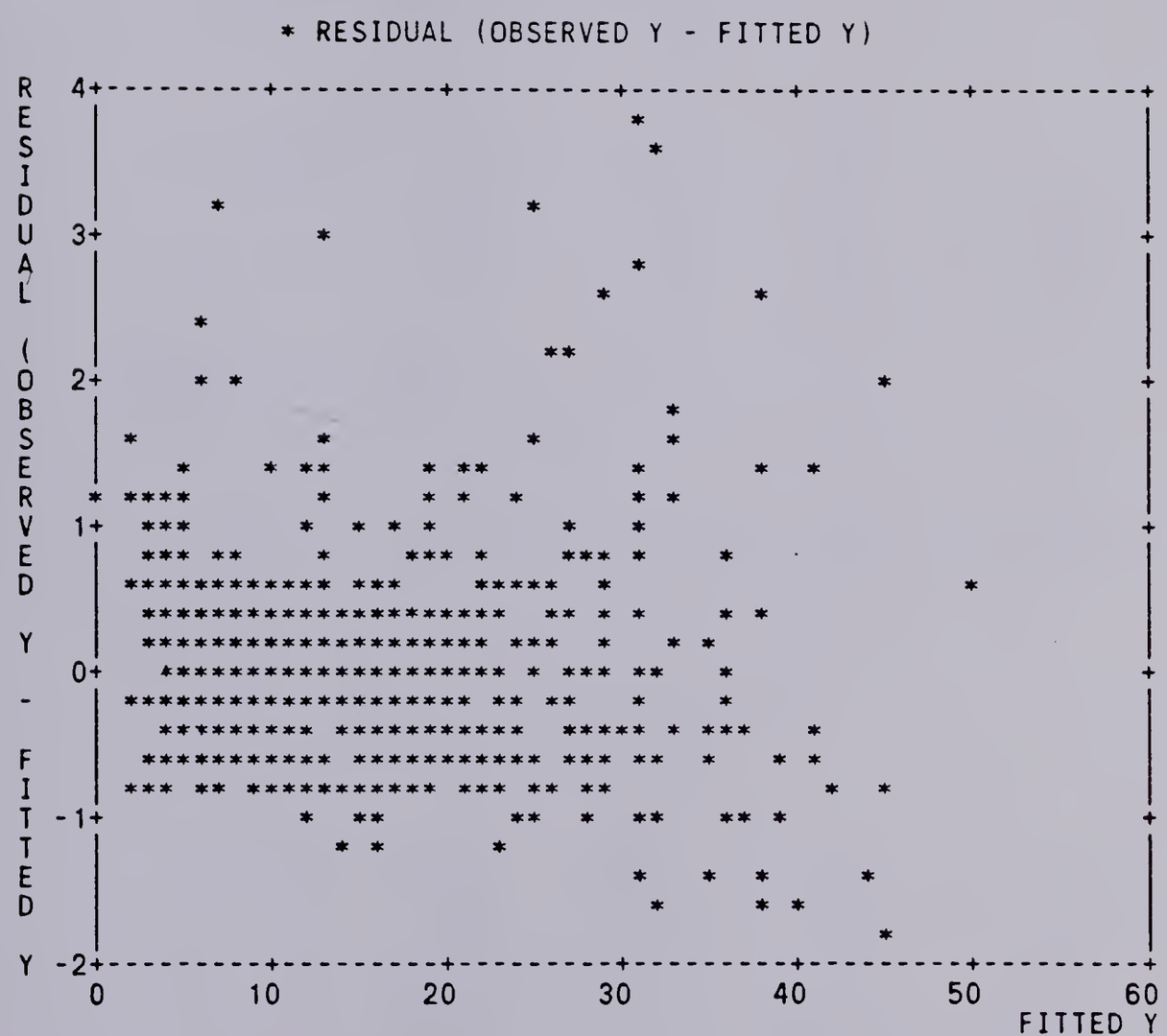
Figure 1 Histogram of Residuals

CLASS INTVLS		F	CF	PF	CPF
X	-1.784	1	1	.1	.1
-1.784<X	-1.505	3	4	.4	.5
-1.505<X	-1.226	6	10	.8	1.3
-1.226<X	-.948	13	23	1.7	3.0
-.948<X	-.669	62	85	8.0	11.0
-.669<X	-.391	100	185	12.9	23.9
-.391<X	-.112	177	362	22.9	46.8
-.112<X	.167	156	518	20.2	66.9
.167<X	.445	106	624	13.7	80.6
.445<X	.724	72	696	9.3	89.9
.724<X	1.002	27	723	3.5	93.4
1.002<X	1.281	20	743	2.6	96.0
1.281<X	1.560	11	754	1.4	97.4
1.560<X	1.838	4	758	.5	97.9
1.838<X	2.117	5	763	.6	98.6
2.117<X	2.395	3	766	.4	99.0
2.395<X	2.674	2	768	.3	99.2
2.674<X	2.952	2	770	.3	99.5
2.952<X	3.231	2	772	.3	99.7
3.231<X	3.510	0	772	.0	99.7
3.510<X		2	774	.3	100.0



LEGEND: F:FREQUENCY, CF:CUMULATIVE FREQUENCY
PF:PERCENT FREQUENCY, CPF:CUMULATIVE PERCENT FREQUENCY

Figure 2 Plot of Residuals



APPENDIX IV - Example of a BATCH Run of the Aspen Growth Model.

```
**********************************************************
********** ASPEN STAND DEVELOPMENT MODEL ************
**********       RUN: 1981 3 19 12          ************
**********************************************************

STAND AGE IS 14 YEARS
GROWING PERIOD 0 OF MAXIMUM 2
LENGTH OF GROWING PERIOD IS 5 YEARS
PLOT SIZE IS 405 SQUARE METERS
SITE INDEX IS 16 ; INDEX AGE 50
```

2 CM. DBH CLASS	TOTAL VOLUME	TOTAL BIOMASS	TOTAL BA	TALLY
0 - 2	1.067	.902	.460	2296
2 - 4	13.953	10.878	6.013	8469
4 - 6	13.903	10.323	5.992	3704
6 - 8	.335	.240	.144	49
TOTALS	29.257	22.343	12.609	14519

```
AVERAGE DIAMETER (CM) :      3.144
AVERAGE HEIGHT (M) :         5.456
BIOMASS MEAN ANNUAL INCREMENT (TONNES PER HA):    1.596
```

```
**********************************************************
********** ASPEN STAND DEVELOPMENT MODEL ************
*******       RUN: 1981 3 19 12          ************
**********************************************************

STAND AGE IS 19 YEARS
GROWING PERIOD 1 OF MAXIMUM 2
LENGTH OF GROWING PERIOD IS 5 YEARS
PLOT SIZE IS 405 SQUARE METERS
SITE INDEX IS 16 ; INDEX AGE 50
```

2 CM. DBH CLASS	TOTAL VOLUME	TOTAL BIOMASS	TOTAL BA	TALLY
0 - 2	.020	.016	.006	25
2 - 4	7.068	5.314	2.293	2494
4 - 6	31.879	22.851	10.341	5185
6 - 8	11.941	8.301	3.874	1185
TOTALS	50.908	36.481	16.514	8889

```
AVERAGE DIAMETER (CM) :      4.738
AVERAGE HEIGHT (M) :         7.218
BIOMASS MEAN ANNUAL INCREMENT (TONNES PER HA):    1.920
```

```
**********************************************************
********** ASPEN STAND DEVELOPMENT MODEL ************
**********       RUN: 1981 3 19 12         ************
**********************************************************

STAND AGE IS 24 YEARS
GROWING PERIOD 2 OF MAXIMUM 2
LENGTH OF GROWING PERIOD IS 5 YEARS
PLOT SIZE IS 405 SQUARE METERS
SITE INDEX IS 16 ; INDEX AGE 50
```

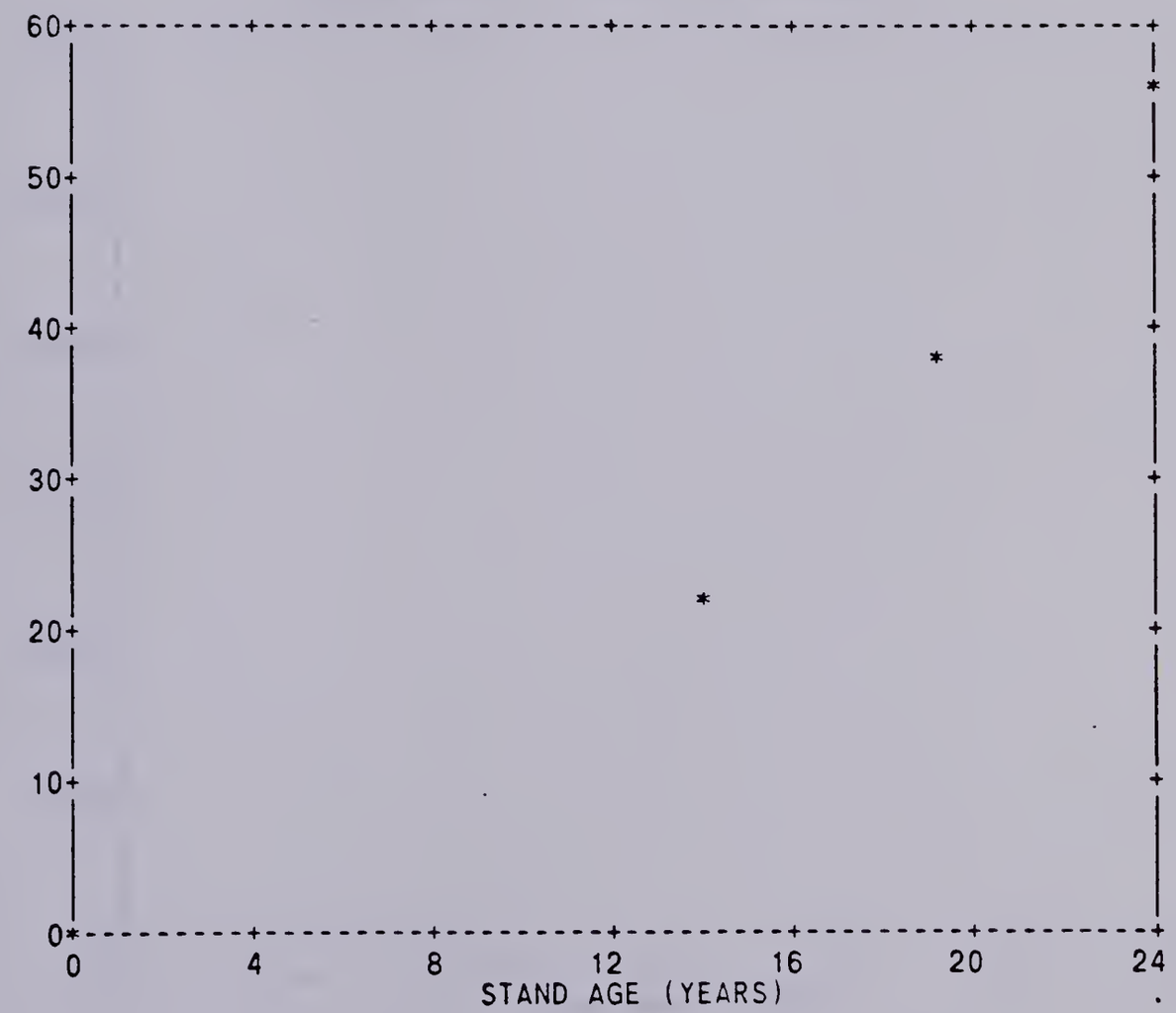
2 CM. DBH CLASS	TOTAL VOLUME	TOTAL BIOMASS	TOTAL BA	TALLY
2 - 4	.233	.169	.061	49
4 - 6	15.767	11.033	4.139	1852
6 - 8	54.241	36.843	14.238	3975
8 - 10	8.937	5.905	2.346	420
TOTALS	79.177	53.949	20.784	6296

```
AVERAGE DIAMETER (CM) :      6.406
AVERAGE HEIGHT (M) :         8.884
BIOMASS MEAN ANNUAL INCREMENT (TONNES PER HA):    2.248
```

ASPEN STAND DEVELOPMENT MODEL 1981 2 23 17

STAND BIOMASS VS. STAND AGE

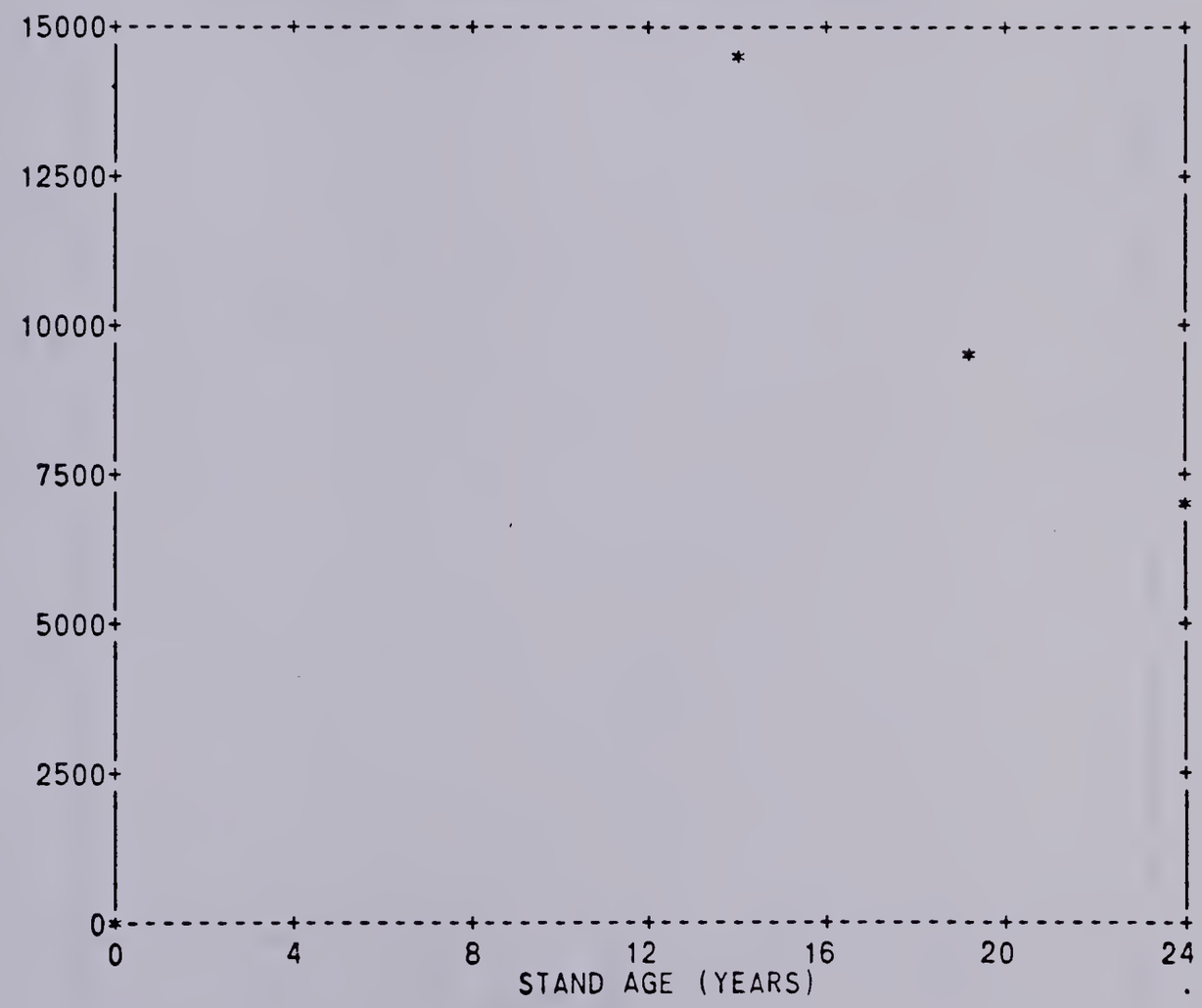
* TOTAL STAND BIOMASS (TONNES PER HECTARE)



ASPEN STAND DEVELOPMENT MODEL 1981 2 23 17

STAND DENSITY VS. STAND AGE

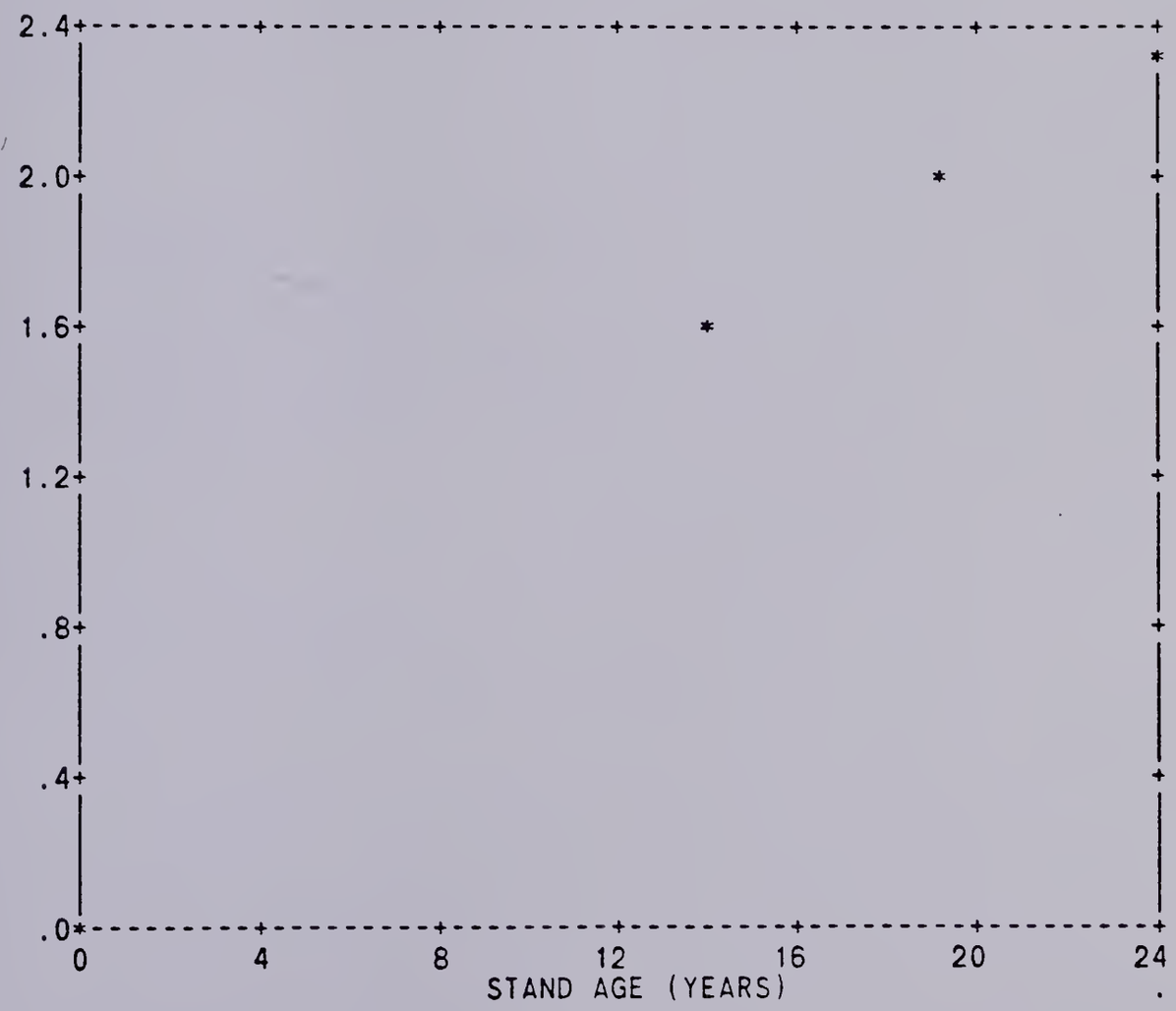
* STAND DENSITY (TREES PER HECTARE)



ASPEN STAND DEVELOPMENT MODEL 1981 2 23 17

MEAN ANNUAL INCREMENT VS. STAND AGE

* MEAN ANNUAL INCREMENT (TONNES PER HECTARE)



APPENDIX V - Alphabetical Listing of Functions Used in the Aspen Growth Model

```
**** AVE                                                    AVE

  AVE:(+/ω)÷ρω
     :0=ρρω
     :ω
⍝
⍝  ARITHMETIC MEAN OF VECTOR ω
⍝  IF ω IS A SCALAR, THEN MEAN IS ω
⍝


**** BIOM                                                   BIOM

 BIOM:1E¯6×*¯.80319 + .936736×⊛100×ω×⍺*2
⍝ INDIVIDUAL TREE BIOMASS (TONNES) FROM BELLA AND DE FRANCHESCHI,
⍝ 1980 (PAGE 8).
⍝ ⍺:DBH IN CM.
⍝ ω:HEIGHT IN METERS.


***** BY                                                    BY

  BY  :(((1 0 ×ρω)⌈ρ⍺)↑⍺),((1 0×ρ⍺)⌈ρω)↑ω
      :(0=0\0/,⍺←COL ⍺)≠(0=0\0/,ω←COLω)
      :(⍪⍺) BY (⍪ω)
⍝
⍝  CREATE A MATRIX FROM ⍺ AND ω
⍝  BY FIRST CONVERTING THEM TO MATRICES AND THEN
⍝  ADJUSTING THEIR ROW  SIZES TO MATCH
⍝  AND PLACING ⍺ TO LEFT OF ω.
⍝  IF THEY DIFFER IN TYPE THEY ARE CONVERTED
⍝  TO CHARACTER FORM.


**** CDTA                                                   CDTA

  CDTA:○(ω÷2)*2
⍝
⍝   CIRCLE DIAMETER TO AREA
⍝   RESULT IS AREA OF CIRCLE WITH DIAMETER ω
⍝
```

```
****  CLASS                                                    CLASS

    ∇ OUT←DCLASS CLASS DWD;UPCLASS;LOCLASS

 [1]  UPCLASS←⌈NUB DCLASS×DWD

 [2]  LOCLASS←UPCLASS-DWD

 [3]  OUT←(9 0 ▼COL LOCLASS) BY(COL((⍴LOCLASS),3)⍴(' - ')) BY(2 0 ▼COL UPCLA
      SS)
 [4]  ⍝ RESULT IS COLUMN SHOWING DIAMETER CLASS INTERVALS.

 [5]  ⍝ DCLASS : VECTOR OF DIAMETER CLASSES

 [6]  ⍝ DWD : WIDTH OF CLASS

 [7]  ⍝

****  COL                                                      COL

  COL:(2↑(⍴⍵),1 1)⍴⍵
⍝
⍝ RESTRUCTURE ⍵ AS MATRIX WITH AT LEAST ONE COLUMN.
⍝   (ONLY FIRST TWO COORDINATES OF STRUCTURE ARE
⍝      RETAINED.)
⍝

****  DBHTODCL                                                 DBHTODCL

    ∇ R←V DBHTODCL S
 [1]  ⍝ ASSIGNMENT OF A TREE DIAMETER TO A DIAMETER CLASS.
 [2]  ⍝ V IS A VECTOR
 [3]  ⍝ S[1]=DMIN, S[2]=DWD, S[3]=DNUM
 [4]  R←⌈0.01+(V-S[1])÷S[2]
 [5]  R[(R≥S[3])/⍳(⍴V)]←S[3]

****  DIST                                                     DIST

 DIST:(⌽DIS ⍵[;1])+.× ⌽ ⍵[;2]
⍝
⍝ RESULT IS ⍵[;2] DISTRIBUTED INTO A ONE WAY TABLE.
⍝ MULTIPLE ENTRIES INTO THE SAME CELL ARE SUMMED
⍝
⍝  USUALLY ⍵[;1] IS DBH CLASS
⍝          ⍵[;2] IS STAND CHARACTERISTIC, SUCH AS VOLUME
⍝
```

```
****  FDBH                                                    FDBH

 FDBH:BFDBH+.×⌽⍺[1],⍺[7],(((+/⍵=⍵)×1E4÷⍺[2])*2),⍵ BY (REL ⍵) BY (CDTA ⍵)
⍝
⍝ THIS FUNCTION CALCULATES 5 YEAR FUTURE DBH AS A FUNCTION OF
⍝ STAND AGE, SITE INDEX, (NUMBER OF TREES PER HECTARE)*2,
⍝  PRESENT DBH(CM.), RELATIVE DBH AND BASAL AREA(CM.*2) OF THE TREE.
⍝ ⍺[1]:STAND AGE (YEARS)
⍝ ⍺[7]:SITE INDEX (M*2 AT 50 YEARS)
⍝ ⍵:A VECTOR OF ALL DIAMETERS IN THE PLOT.
⍝ SOURCE: RUN 1, JANUARY 26, 1981, FRST.REG VERSION.
⍝ /
⍝_____1981 2 8

****  HGT                                                     HGT

  HGT:.011292 .440716 ¯.002036 LTR ⍵*0 1 2
⍝ AVERAGE HEIGHT OF ASPEN STAND IN METERS, ⍵: STAND AGE.
⍝ FROM 1980 ENFOR REPORT.
```

```
**** LTR                                                    LTR

  LTR:⍺+.×⍵
⍝ LINEAR TRANSFORMATION, OR PRODUCT SUM OF ⍺ WITH ⍵

**** MAI                                                    MAI

 MAI:⍵+⍺
⍝ ⍺:A VECTOR OF TIMES SINCE BEGINNING
⍝ ⍵:VECTOR OF SOME GROWTH PARAMETER OVER TIME.

**** MAIN                                                   MAIN

     ∇ OUT←STANDCHAR MAIN DBH;COUNT;DBHQ

 [ 1]  COUNTQ←0

 [ 2]  TOTAL← 0 4 ⍴⍳0

 [ 3]  STANDCHAR←STANDCHAR,0

 [ 4]  REPEAT:

 [ 5]  TALLY←(⍴DBH)⍴10000÷STANDCHAR[2]

 [ 6]  VOLUME←TALLY×DBH VOL STANDCHAR[1] STH STANDCHAR[7]

 [ 7]  BIOMASS←TALLY×DBH BIOM STANDCHAR[1] STH STANDCHAR[7]

 [ 8]  BA←(CDTA DBH)÷STANDCHAR[2]

 [ 9]  STANDCHAR REPORT DBH

 [10]  TOTAL←TOTAL ON1 STANDCHAR[1],(+⌿BIOMASS BY TALLY),(+/BIOMASS)÷STANDCH
    AR[1]
 [11]  DBH←(STANDCHAR MORT DBH)/STANDCHAR FDBH DBH

 [12]  STANDCHAR[1]←STANDCHAR[1]+5

 [13]  STANDCHAR[8]←STANDCHAR[8]+1

 [14]  →REPEAT IF STANDCHAR[8]≤STANDCHAR[3]

 [15]  MODELPLOT TOTAL

 [16]  ⍝_____1981 2 25
```

```
****  MODELPLOT                                                    MODELPLOT

     ∇ MODELPLOT TOTAL
 [ 1]  QYLABEL←'TOTAL STAND BIOMASS (TONNES PER HECTARE)'
 [ 2]  QYCOLABEL←''
 [ 3]  QTITLE←'STAND BIOMASS VS. STAND AGE'
 [ 4]  1 2 PLT TOTAL
 [ 5]  QYLABEL←'STAND DENSITY (TREES PER HECTARE)'
 [ 6]  QYCOLABEL←''
 [ 7]  QTITLE←'STAND DENSITY VS. STAND AGE'
 [ 8]  1 3 PLT TOTAL
 [ 9]  QYLABEL←'MEAN ANNUAL INCREMENT (TONNES PER HECTARE)'
 [10]  QYCOLABEL←''
 [11]  QTITLE←'MEAN ANNUAL INCREMENT VS. STAND AGE'
 [12]  1 4 PLT TOTAL


****  MORT                                                         MORT

   / ∇ OUT←STANDCHAR MORT DBH;RANDOM;NUM;PMORT;STANDCHAR
 [ 1]  NUM←⍴DBH
 [ 2]  RANDOM←(NUM,5)⍴(?(NUM×5)⍴1000)÷1000
 [ 3]  OUT←(∧/RANDOM≥⍉(5,NUM)⍴PMORT←STANDCHAR SELECT DBH)
 [ 4]  ⍝ THE LENGTH OF THE TREE LIST (DBH) VECTOR IS RECORDED.
 [ 5]  ⍝ A MATRIX OF RANDOM NUMBERS IS THEN GENERATED, SHAPED
 [ 6]  ⍝ SO THAT THERE ARE 5 RANDOM NUMBERS FOR EACH TREE.
 [ 7]  ⍝ IF THE PROBABILITY OF MORTALITY FOR A TREE IS LESS
 [ 8]  ⍝ THAN ALL 5 RANDOM NUMBERS (REPRESENTING 5 ANNUAL
 [ 9]  ⍝ CHANCES TO DIE), THAT TREE IS MAINTAINED FOR THE
 [10]  ⍝ FOLLOWING GROWTH PERIOD, OTHERWISE IT 'DIES' AND
 [11]  ⍝ IS OMMITTED FROM THE TREE LIST VECTOR IN THE
 [12]  ⍝ FUNCTION 'MAIN'.


****  ODIS                                                         ODIS

  ODIS: (ONUB ⍵)∘.=⍵
⍝
⍝  ORDERED DISTRIBUTION OF ⍵ OTHERWISE SAME AS DIS
⍝


****  ONUB                                                         ONUB

  ONUB : ((⍵⍳⍵)=⍳⍴⍵)/⍵←⍵[⍋⍵]
⍝
⍝ UNIQUE ELEMENTS OF NUMERIC VECTOR ⍵
⍝   IN ASCENDING ORDER.
⍝ (ORDERED NUB)
⍝
```

```
****  ON1                                                          ON1

 ON1  :(((0 1 ×ρω)⌈ρα)↑α),[⎕IO]((0 1×ρα)⌈ρω)↑ω
      :(0=0\0/,α←ROW α)≠(0=0\0/,ω←ROWω)
      :(▼α) ON1 (▼ω)
⍝
⍝  CREATE A MATRIX FROM α AND ω
⍝  BY FIRST CONVERTING THEM TO MATRICES AND THEN
⍝  ADJUSTING THEIR COLUMN SIZES TO MATCH
⍝  AND PLACING α ON TOP OF ω.
⍝  IF THEY DIFFER IN TYPE THEY ARE CONVERTED
⍝  TO CHARACTER FORM.
⍝


****  PKILL                                                        PKILL

 PKILL:+1+×-B+.×⍉1,α[1],((+/BA)÷α[2]),ω BY(REL ω) BY (REL BA) BY BA←CDTA ω
      :((ρ,ω)×1E4÷α[2])≥2500
      :+1+×-BB LTR ⍉1,COL REL BA←CDTA ω
⍝  RESULT IS PROBABILITY OF DEATH FOR ASPEN TREES
⍝  α : TWO ELEMENT VECTOR OF STAND AGE AND PLOT SIZE (M×2)
⍝  ω : VECTOR OF TREE DBH'S (CM) FOR THE PLOT
⍝  IF THE NUMBER OF TREES PER HECTARE IS GREATER THAN 2500,
⍝  THE LOWER FUNCTION IS USED.
⍝  TOP FUNCTION : RUN 1, FEBRUARY 8, 1981
⍝  BOTTOM FUNCTION : RUN 2, JANUARY 12, 1981
⍝_____________________1981 2 8


****  REL                                                          REL

  REL:ω + AVE ω
⍝ THIS FUNCTION CALCULATES ANY RELATIVE STAND STATISTIC
```

```
****  REPORT                                                    REPORT
     ∇ STANDCHAR REPORT DBH;DCLASS;VOLUMES;TALLYS;BASALS;CLASSES;BIOMS
 [ 1]  ''
 [ 2]  '       ********************************************************* '
 [ 3]  '       ********** ASPEN STAND DEVELOPMENT MODEL ************ '
 [ 4]  '       **********      RUN: ',(▼□TS[⍳4]),'         ************ '
 [ 5]  '       ********************************************************* '
 [ 6]  ''
 [ 7]  '       STAND AGE IS ',(▼STANDCHAR[1]),' YEARS'
 [ 8]  '       GROWING PERIOD ',(▼STANDCHAR[8]),' OF MAXIMUM ',▼STANDCHAR[3]
 [ 9]  '       LENGTH OF GROWING PERIOD IS 5 YEARS'
 [10]  '       PLOT SIZE IS ',(▼STANDCHAR[2]),' SQUARE METERS'
 [11]  '       SITE INDEX IS ',(▼STANDCHAR[7]),' ; INDEX AGE 50'
 [12]  ''
 [13]  ''
 [14]  '         ',(2 0 ▼STANDCHAR[5]),' CM.     TOTAL      TOTAL      TOTAL
     TALLY'
 [15]  '       DBH CLASS   VOLUME    BIOMASS        BA '
 [16]  DCLASS←DBH DBHTODCL STANDCHAR[4,5,6]
 [17]  CLASSES←DCLASS CLASS STANDCHAR[5]
 [18]  VOLUMES←(DIST DCLASS BY VOLUME)
 [19]  BASALS←(DIST DCLASS BY BA)
 [20]  BIOMS←(DIST DCLASS BY BIOMASS)
 [21]  TALLYS←(DIST DCLASS BY TALLY)
 [22]  CLASSES BY(11 3 11 3 10 3 12 0 ▼VOLUMES BY BIOMS BY BASALS BY TALLYS)
 [23]  '       TOTALS   ', 9 3 11 3 10 3 12 0 ▼(+/VOLUME),(+/BIOMS),(+/BA),+
    /TALLYS
 [24]  ' '
 [25]  '  AVERAGE DIAMETER (CM) :  ', 8 3 ▼(AVE DBH)
 [26]  '  AVERAGE HEIGHT (M) :     ', 8 3 ▼(STANDCHAR[1] STH STANDCHAR[7])
 [27]  '  BIOMASS MEAN ANNUAL INCREMENT (TONNES PER HA):', 8 3 ▼((+/BIOMS)+S
    TANDCHAR[1])
```

```
[28]  →(COUNTQ=1)/RESET
[29]  COUNTQ←1
[30]  →0
[31]  RESET:
[32]  1
[33]  COUNTQ←0
[34]  →0
[35]  '  ',CLASSES BY(11 3 11 3 10 3 12 0 ▼VOLUMES BY BIOMS BY BASALS BY TA
      LLYS)
```

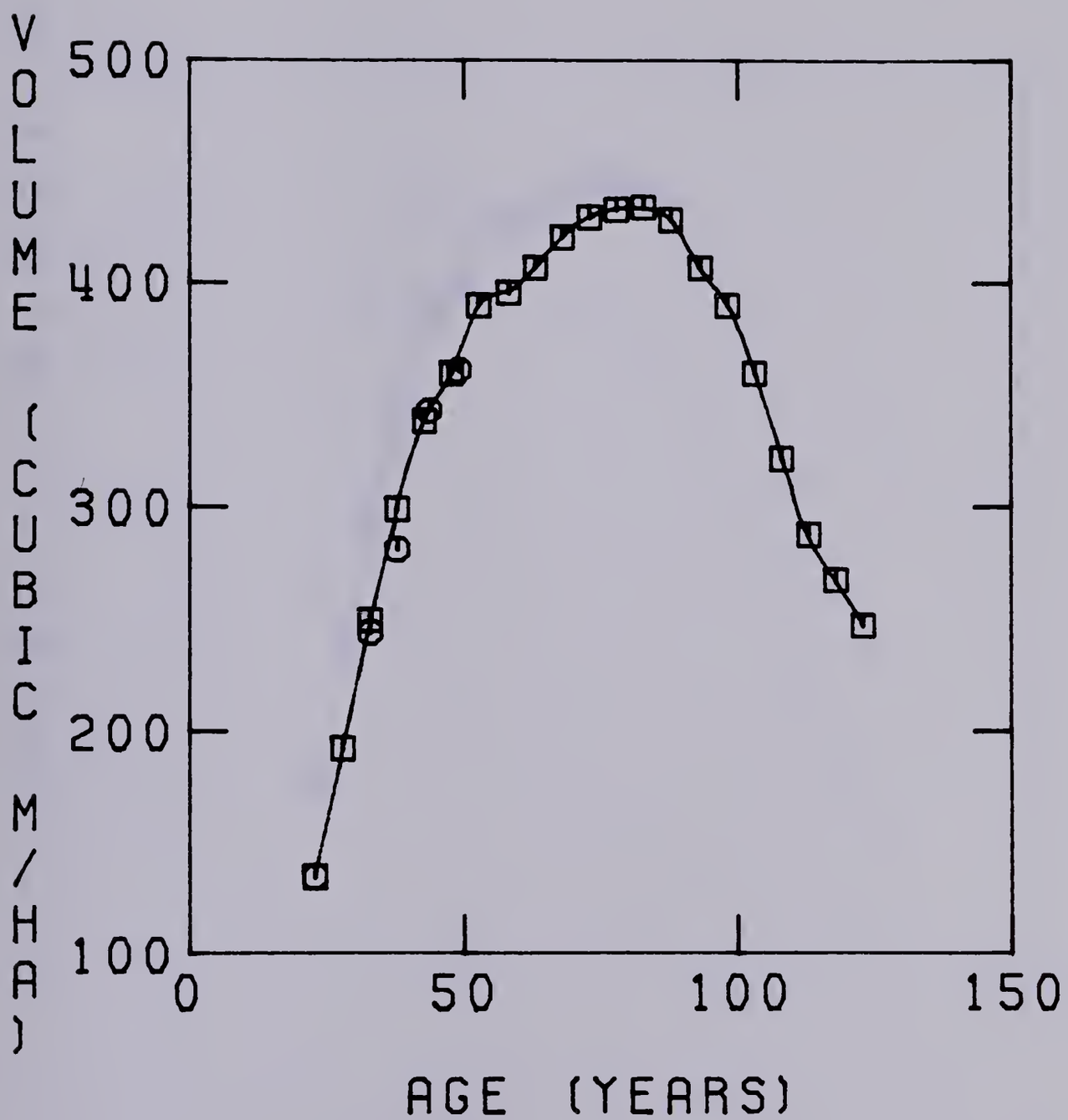
```
**** ROW                                                          ROW

  ROW:(¯2↑1 1,⍴⍵)⍴⍵
⍝
⍝  RESTRUCTURE ⍵ AS MATRIX WITH AT LEAST ONE ROW.
⍝  (ONLY LAST TWO COORDINATES OF STRUCTURE ARE RETAINED.)
⍝


**** SELECT                                                       SELECT

 SELECT:÷1+⍟-BBB LTR ⌽1,COL REL BA←CDTA ⍵
       :((⍴,⍵)×1E4÷⍺[2])<6000
       :⍺ PKILL ⍵
⍝  RESULT IS PROBABILITY OF DEATH FOR ASPEN TREES
⍝  ⍺ : TWO ELEMENT VECTOR OF STAND AGE AND PLOT SIZE (M*2)
⍝  ⍵ : VECTOR OF TREE DBH'S (CM) FOR THE PLOT
⍝  IF THE NUMBER OF TREES PER HECTARE IS GREATER THAN 6000,
⍝  THE TOP FUNCTION IS USED, OTHERWISE, PKILL IS CALLED
⍝  AND FURTHER SELECTION BY DENSITY IS CONDUCTED.
⍝  TOP FUNCTION : RUN 2, JANUARY 21, 1981.
⍝_____________________1981 2 8


**** SPLIT                                                        SPLIT

 SPLIT:(ODIS ⍵[;1])+.× ⌽ ⍵[;2]
⍝
⍝  RESULT IS ⍵[;2] DISTRIBUTED INTO A ONE WAY TABLE.
⍝  MULTIPLE ENTRIES INTO THE SAME CELL ARE SUMMED.
⍝
⍝  USUALLY ⍵[;1] IS DBH CLASS
⍝          ⍵[;2] IS STAND CHARACTERISTIC, SUCH AS VOLUME.
⍝


**** STH                                                          STH

 STH:(HGT ⍺)×⍵÷HGT 50
⍝ SITE TO HEIGHT (PROPORTIONAL), ⍺:AGE, ⍵:SITE INDEX (BASE 50 YEARS)


**** VOL                                                          VOL

  VOL:.028317×((⍺×.3937)*2)÷¯.312+436.683÷⍵×3.28083
⍝ TOTAL VOLUME FOR TREMBLING ASPEN (STUMP AND TOP
⍝ INCLUDED) IN METERS CUBED FROM 'STANDARD VOLUME
⍝ TABLES AND MERCHANTABLE CONVERSION FACTORS FOR THE
⍝ COMMERCIAL TREE SPECIES OF CENTRAL AND EASTERN CANADA',
⍝ BY T.G.HONER, APRIL, 1967. INFORMATION REPORT FMR-X-5.
⍝ ⍺:DBH(CM), ⍵:HEIGHT(M)
```

APPENDIX VI - Graphical Comparison of Actual to Predicted Parameters for Plot 1.

Plot Summary

Plot Size	809 m.
Site Index	21 (m. at 50 years)
Stand Age at Establishment	23 years
Actual Observations	23 to 49 years
Predicted Observations	23 to 123 years
Plot Location	Riding Mountain National Park, Manitoba

Figure 1 Stand Volume ($m.^3$ per ha.)

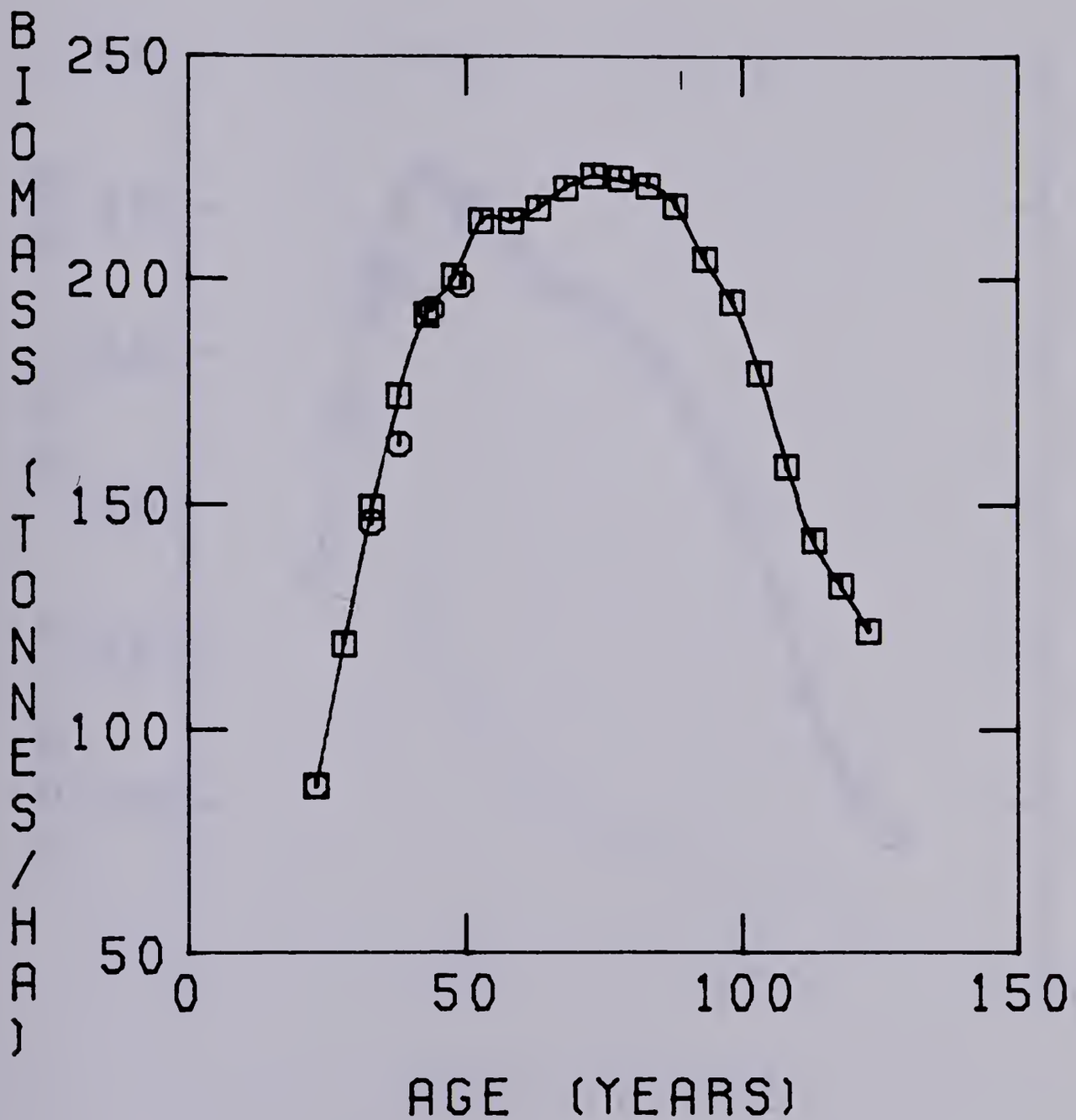


LEGEND: Actual Volume '⊙'

Predicted Volume '□'
(average of 5 runs)

NOTE: Actual Volume = Predicted Volume
at first observation.

Figure 2 Stand Biomass (tonnes per ha.)

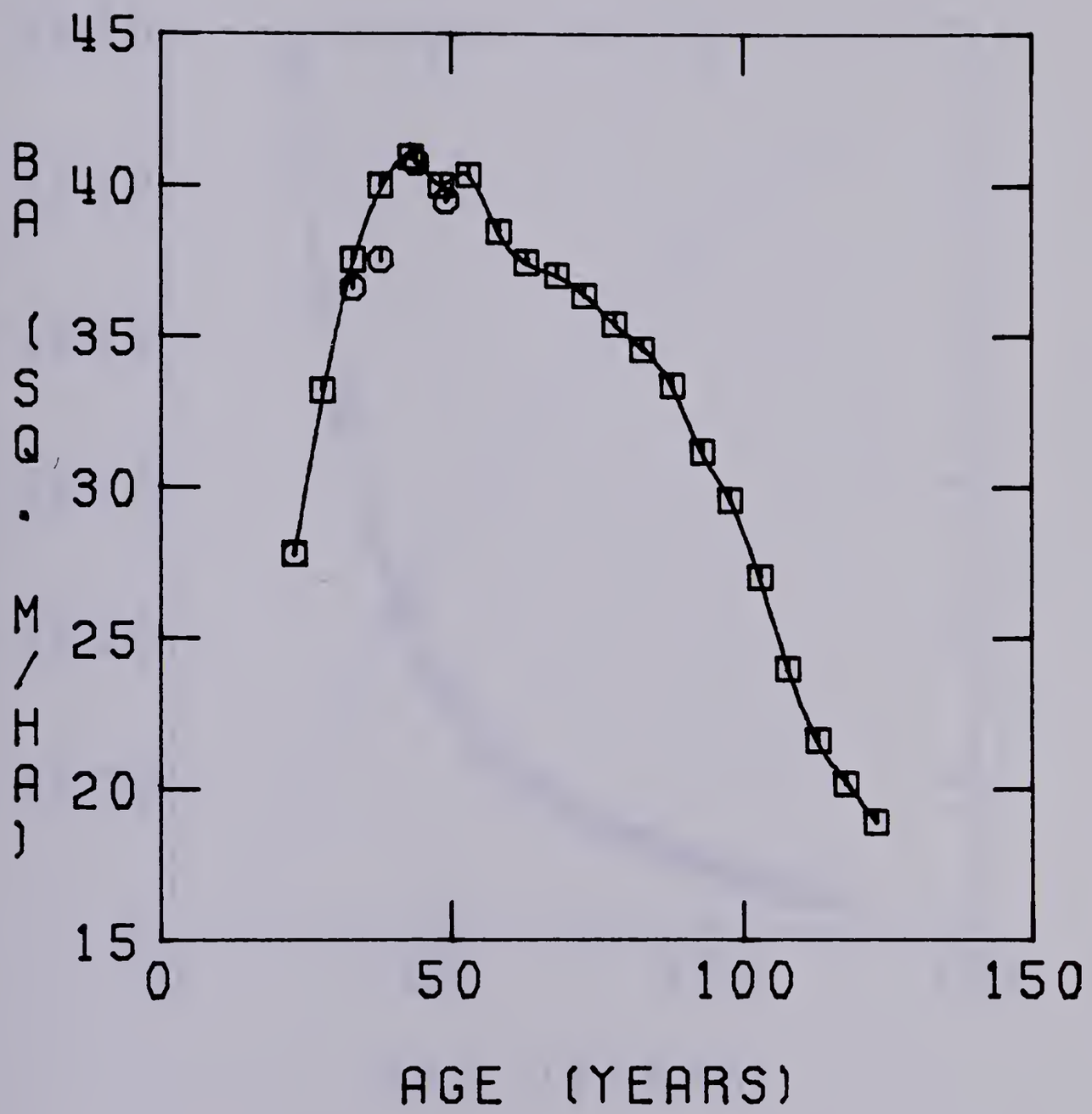


LEGEND: Actual Biomass '⊙'

Predicted Biomass '□'
(average of 5 runs)

NOTE: Actual Biomass = Predicted Biomass
at first observation.

Figure3 Stand Basal Area ($m.^2$ per ha.)

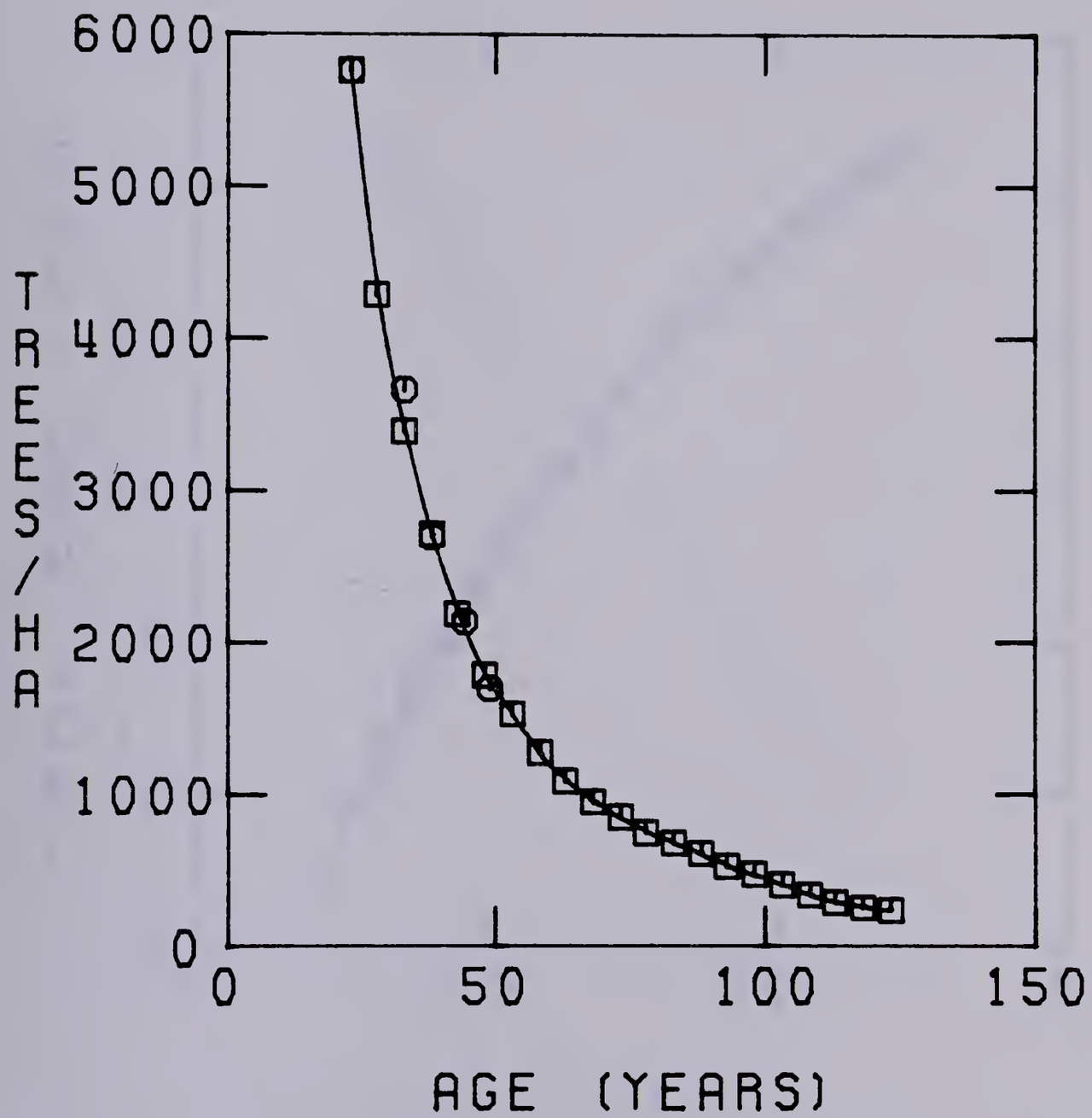


LEGEND: Actual Basal Area '○'

Predicted Basal Area '□'
(average of 5 runs)

NOTE: Actual Basal Area = Predicted Basal Area at first observation.

Figure 4 Stand Density (trees per ha.)

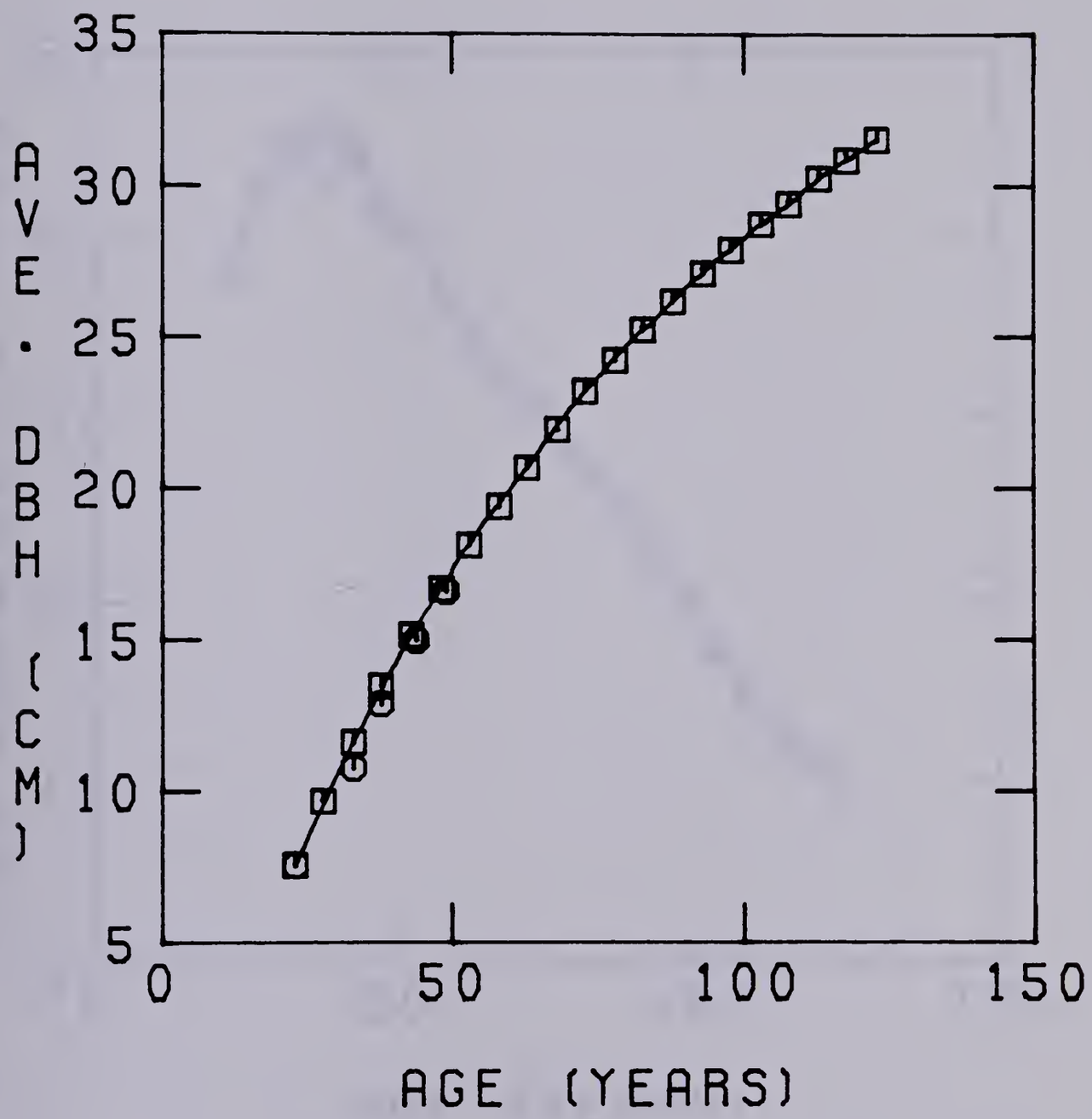


LEGEND: Actual Density 'O'

Predicted Density 'D'
(average of 5 runs)

NOTE: Actual Density = Predicted Density
at first observation.

Figure5 Stand Average DBH (cm.)

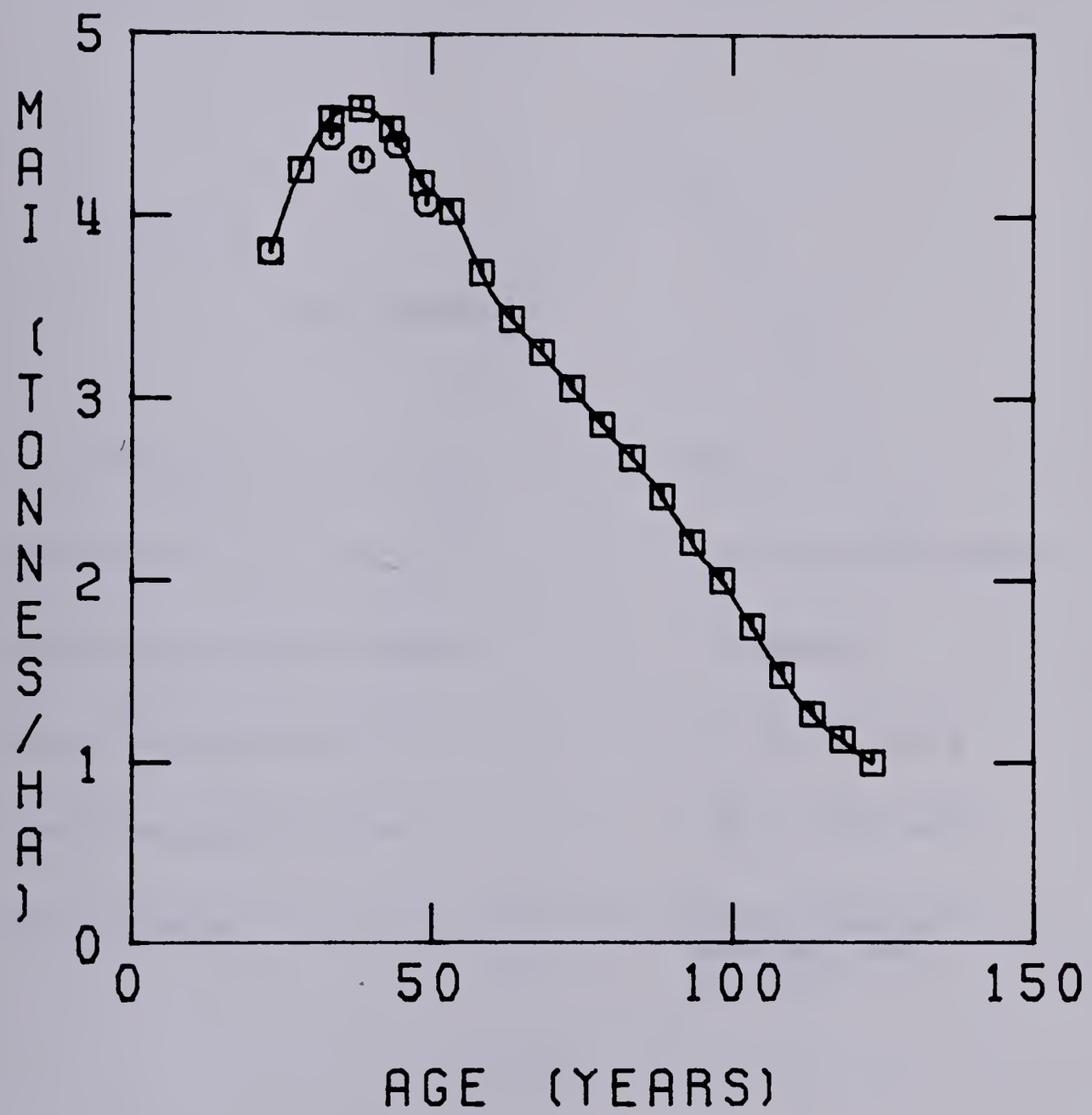


LEGEND: Actual Average DBH '○'

Predicted Average DBH '□'
(average of 5 runs)

NOTE: Actual Average DBH = Predicted Average DBH at first observation.

Figure6 Stand Biomass Mean Annual Increment (tonnes/ha.)



LEGEND: Actual MAI '○'

Predicted MAI '□'
(average of 5 runs)

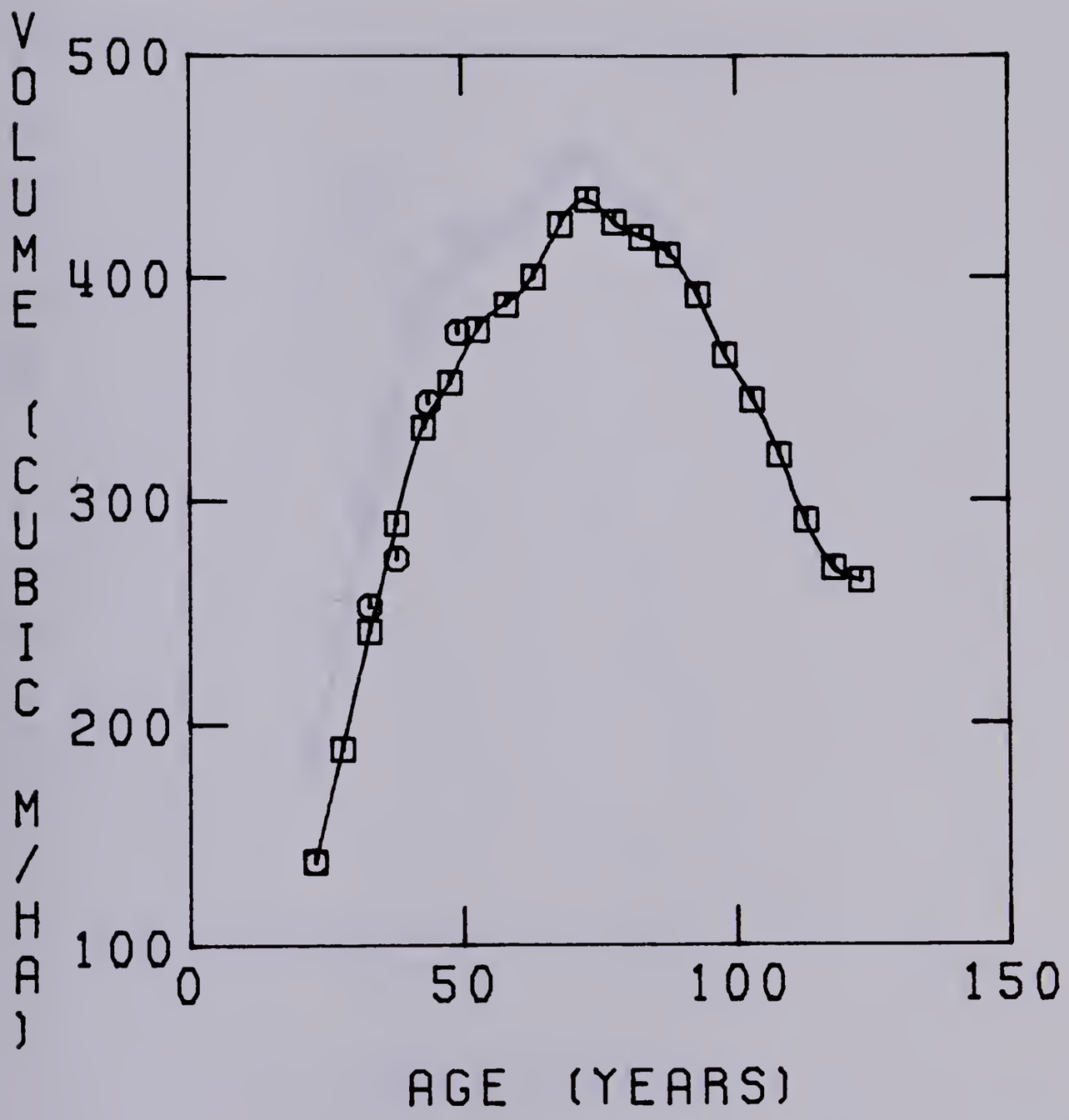
NOTE: Actual MAI = Predicted MAI
at first observation.

APPENDIX VII - Graphical Comparison of Actual to Predicted Parameters for Plot 2.

Plot Summary

Plot Size	809 m.
Site Index	21 (m. at 50 years)
Stand Age at Establishment	23 years
Actual Observations	23 to 49 years
Predicted Observations	23 to 123 years
Plot Location	Riding Mountain National Park, Manitoba

Figure1 Stand Volume ($m.^3$ per ha.)

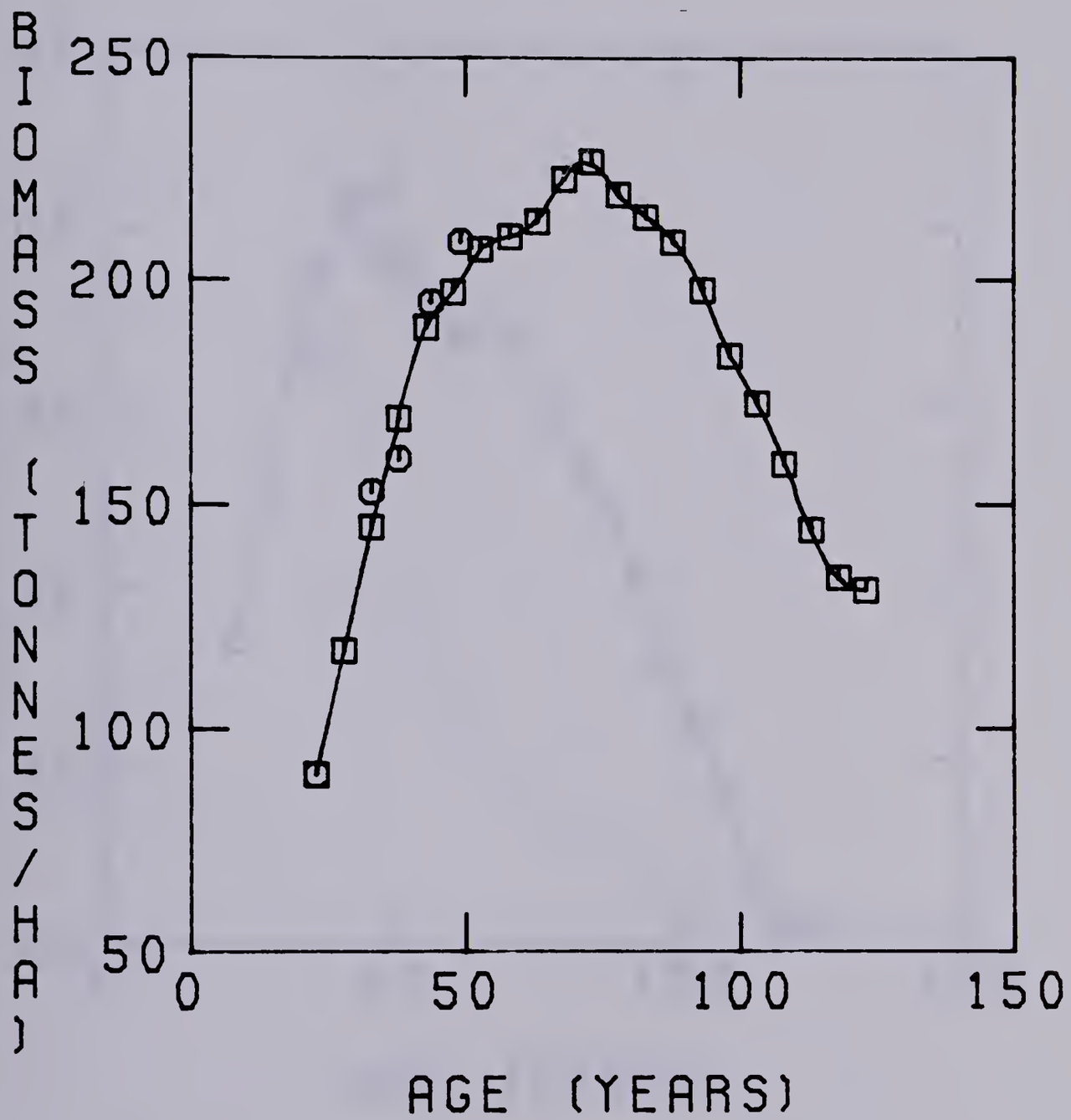


LEGEND: Actual Volume '⊙'

Predicted Volume '□'
(average of 5 runs)

NOTE: Actual Volume = Predicted Volume
at first observation.

Figure 2 Stand Biomass (tonnes per ha.)

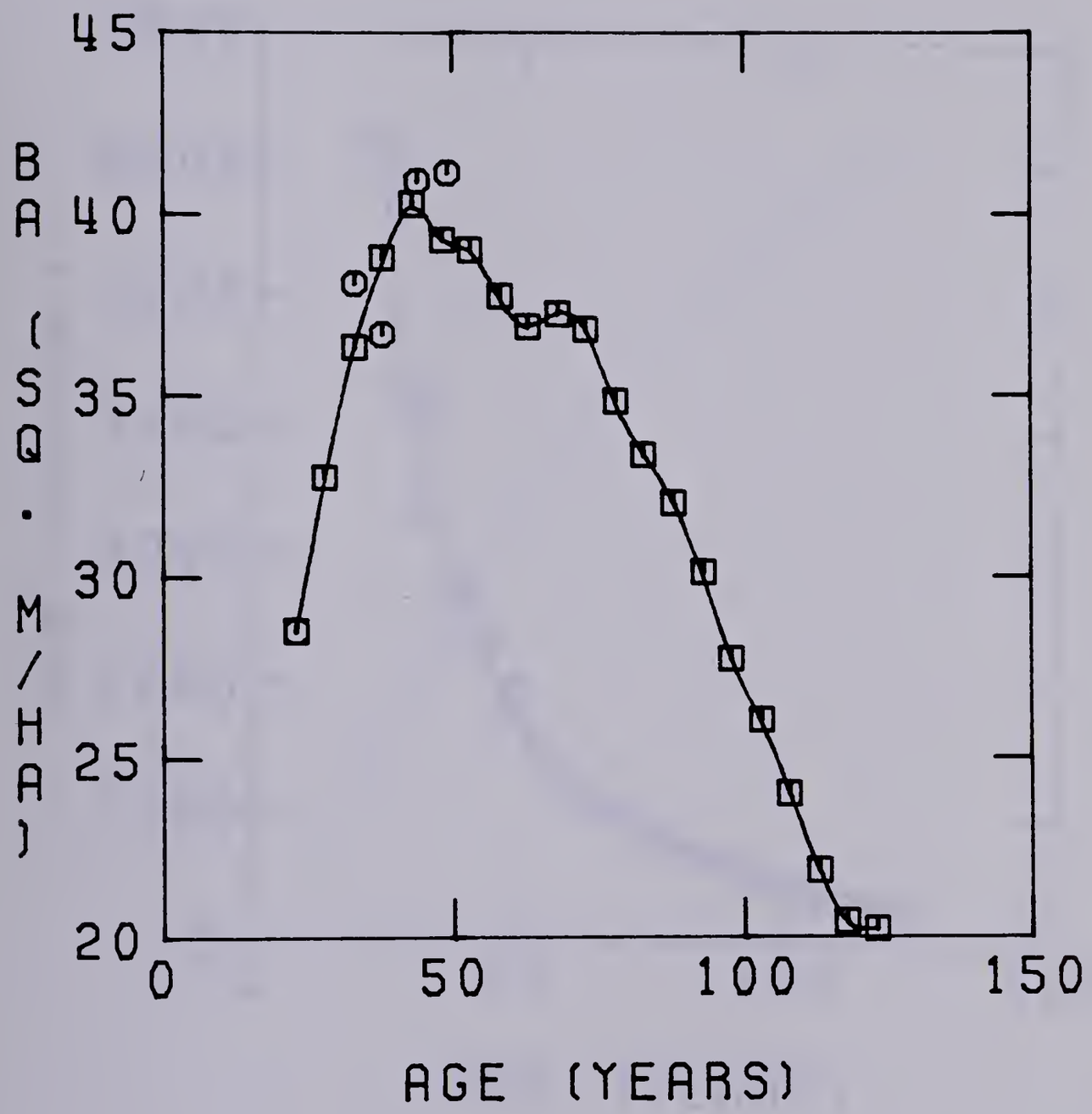


LEGEND: Actual Biomass '⊙'

Predicted Biomass '□'
(average of 5 runs)

NOTE: Actual Biomass = Predicted Biomass
at first observation.

Figure3 Stand Basal Area (m^2 per ha.)

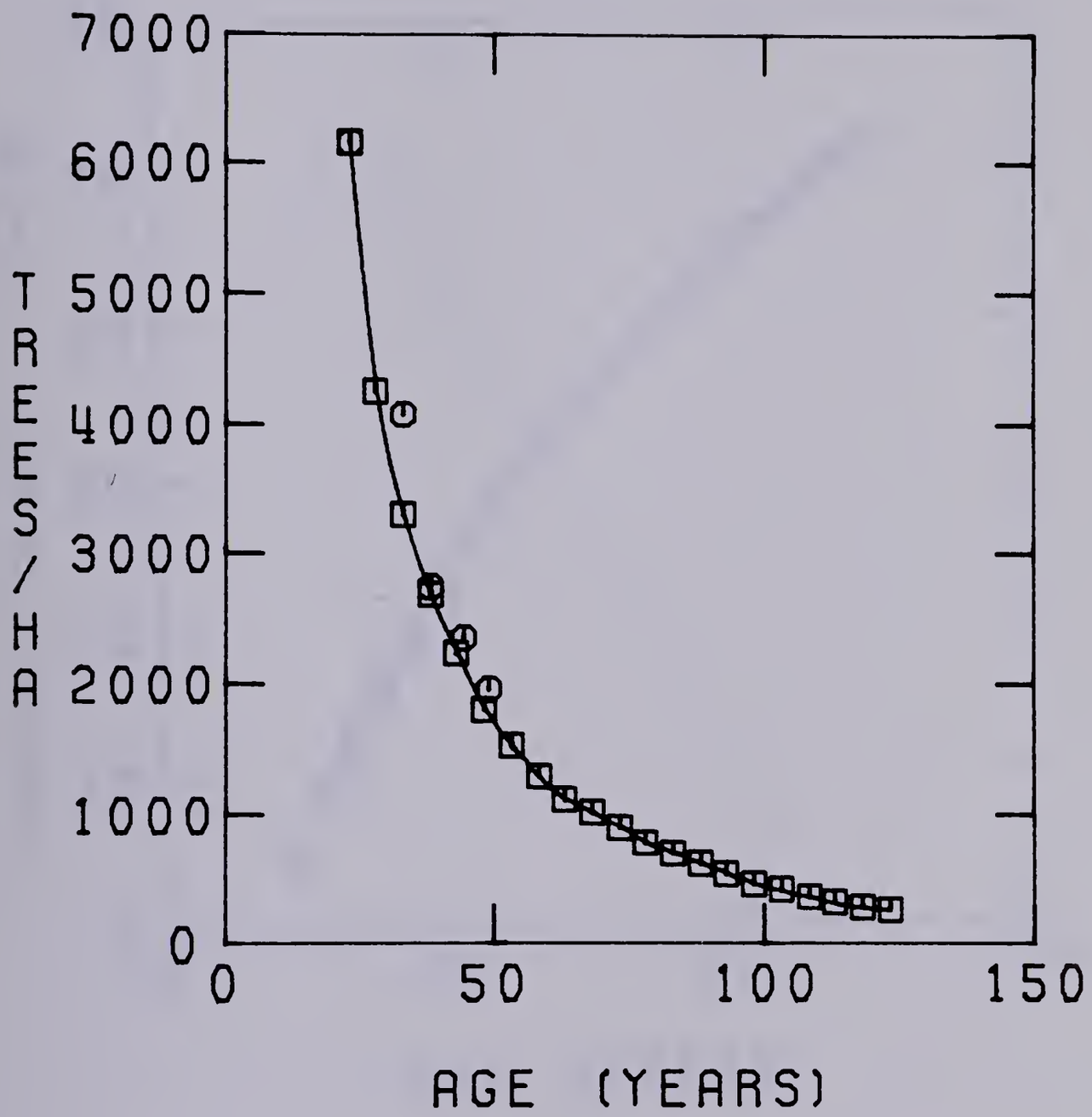


LEGEND: Actual Basal Area '○'

Predicted Basal Area '□'
(average of 5 runs)

NOTE: Actual Basal Area = Predicted Basal Area at first observation.

Figure4 Stand Density (trees per ha.)

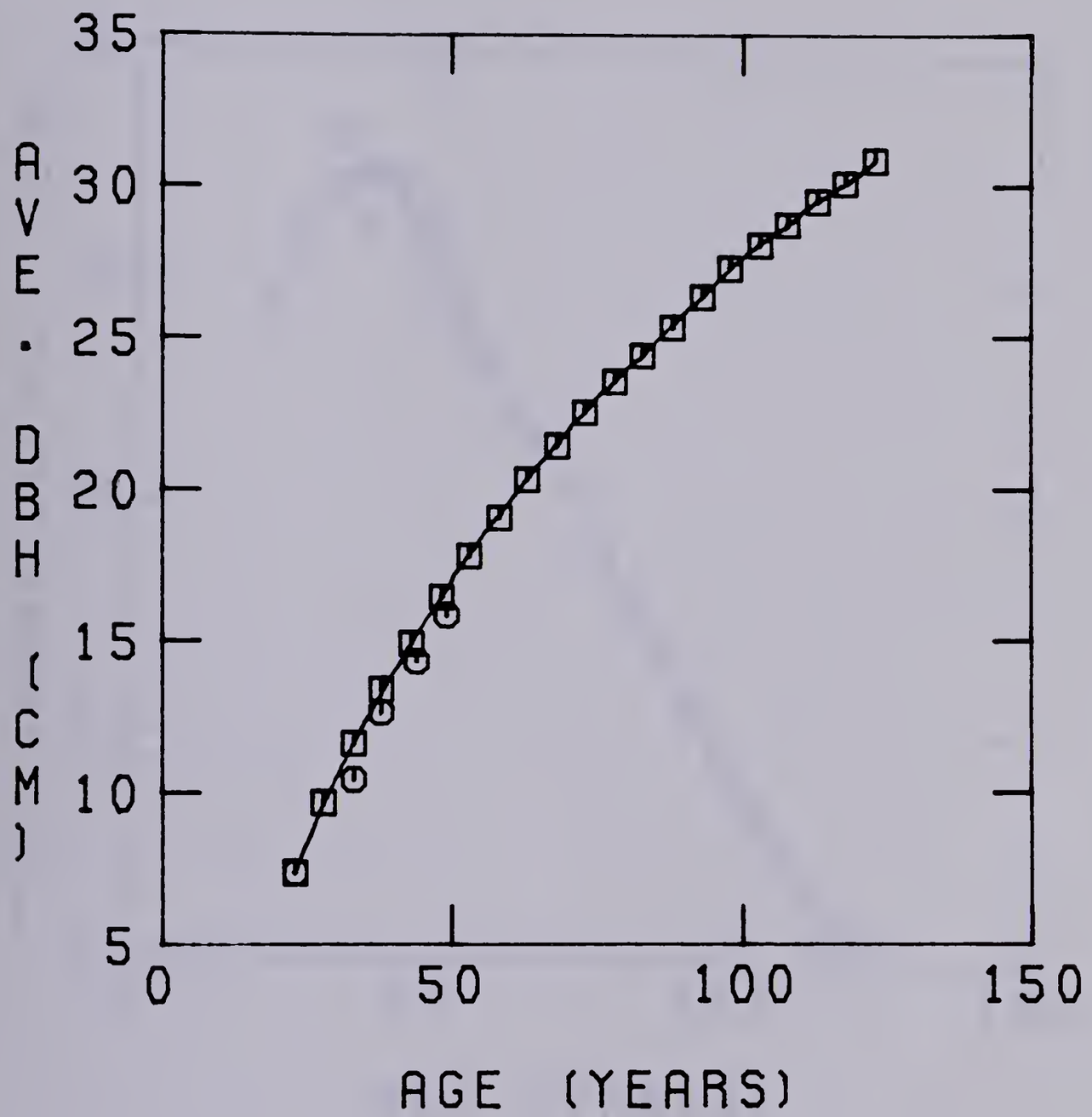


LEGEND: Actual Density '○'

Predicted Density '□'
(average of 5 runs)

NOTE: Actual Density = Predicted Density
at first observation.

Figure 5 Stand Average DBH (cm.)

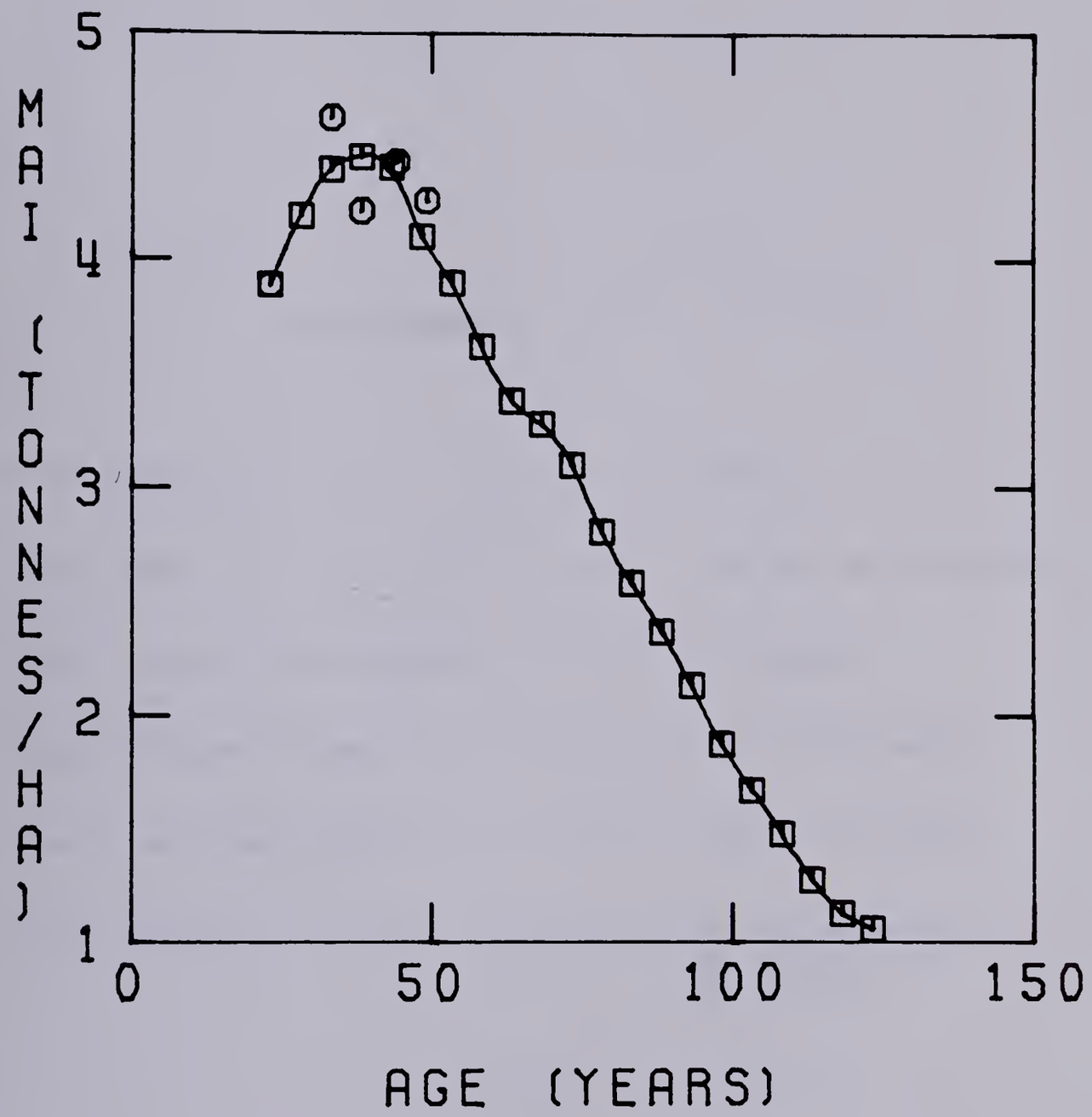


LEGEND: Actual Average DBH '○'

Predicted Average DBH '□'
(average of 5 runs)

NOTE: Actual Average DBH = Predicted Average DBH at first observation.

Figure 6 Stand Biomass Mean Annual Increment (tonnes/ha.)



LEGEND: Actual MAI '○'

Predicted MAI '□'
(average of 5 runs)

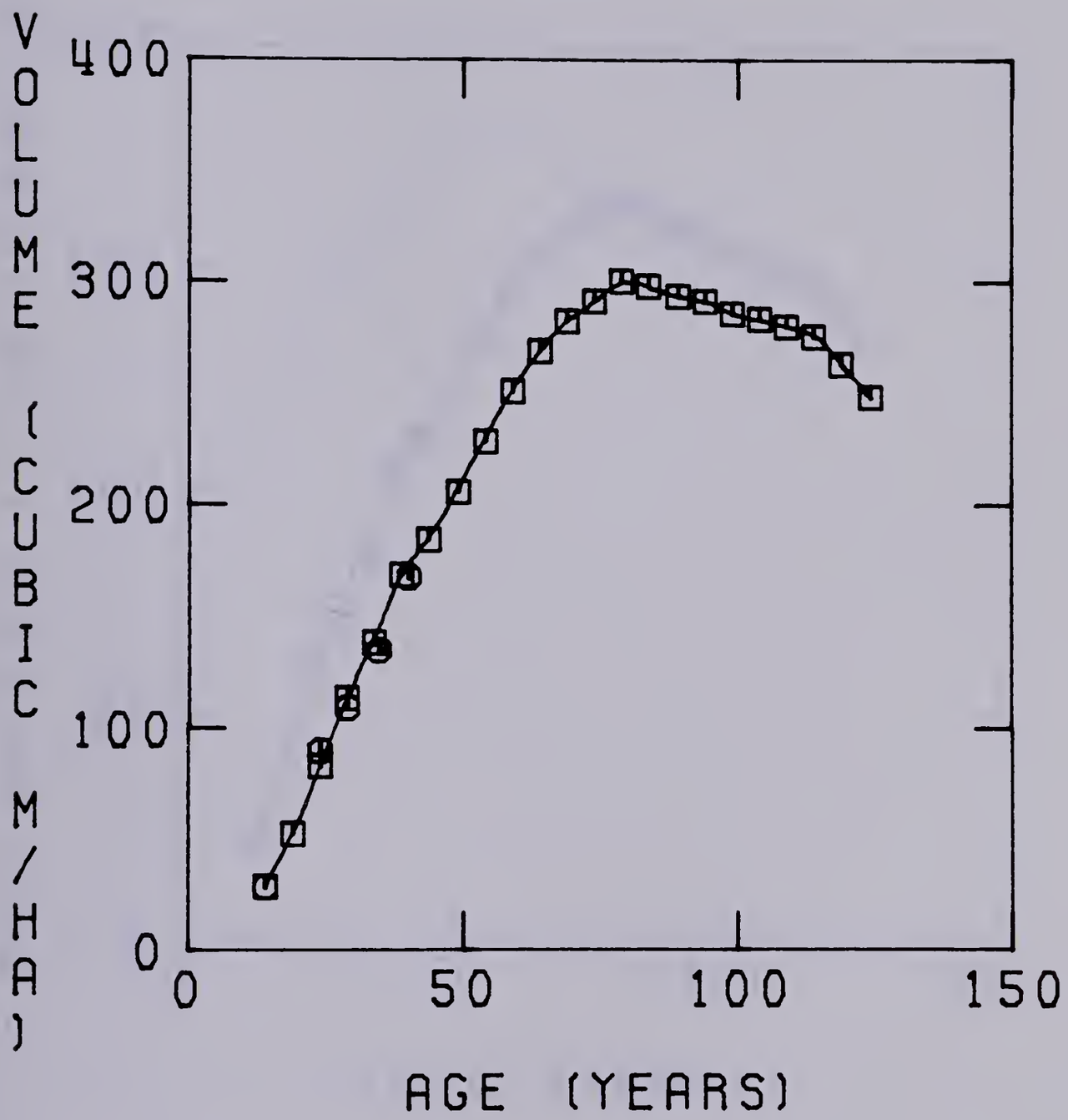
NOTE: Actual MAI = Predicted MAI
at first observation.

APPENDIX VIII - Graphical Comparison of Actual to Predicted Parameters for Plot 3.

Plot Summary

Plot Size	405 m.
Site Index	16 (m. at 50 years)
Stand Age at Establishment	14 years
Actual Observations	23 to 49 years
Predicted Observations	23 to 123 years
Plot Location	Riding Mountain National Park, Manitoba

Figure 1 Stand Volume ($m.^3$ per ha.)

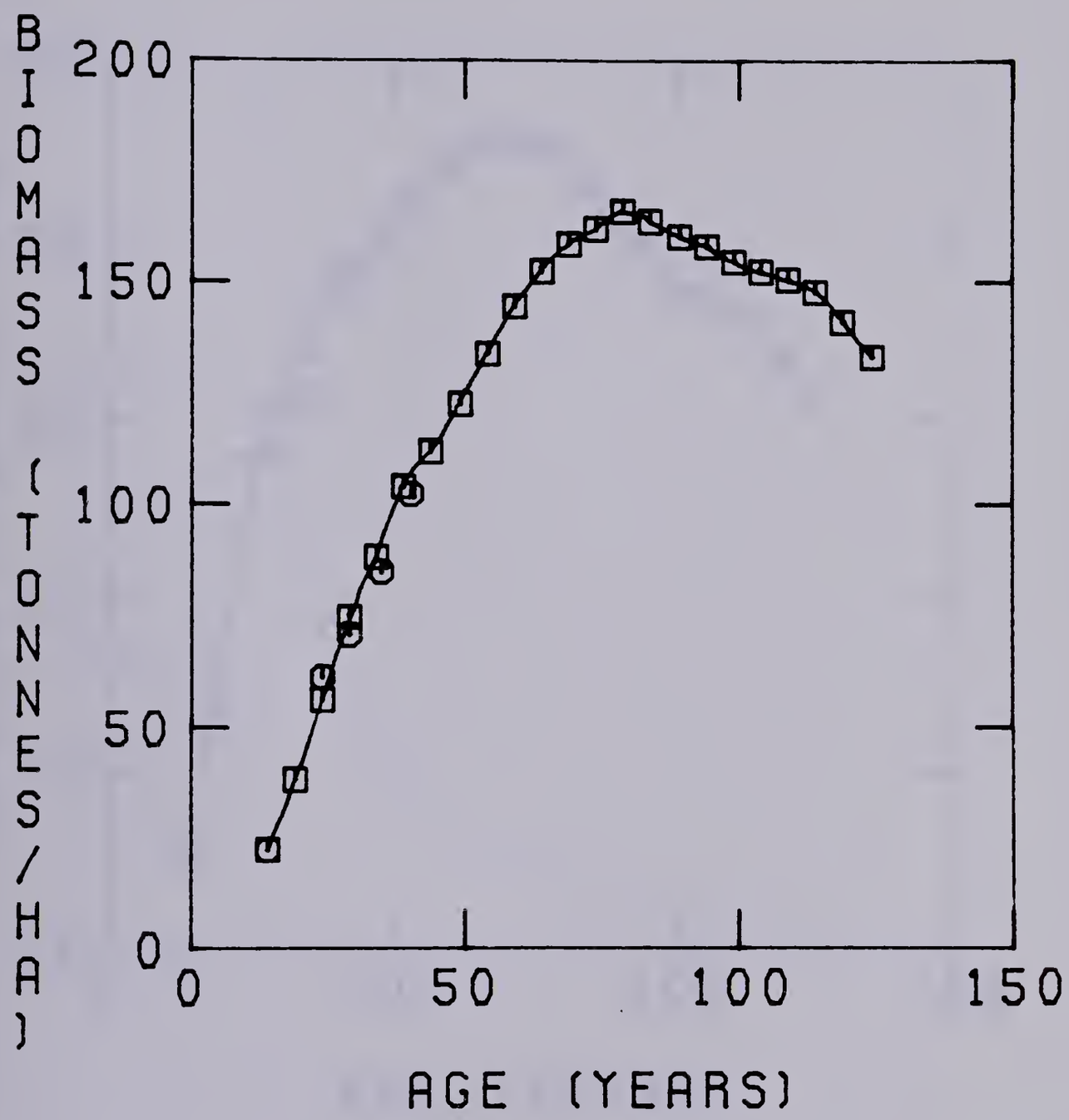


LEGEND: Actual Volume '○'

Predicted Volume '□'
(average of 5 runs)

NOTE: Actual Volume = Predicted Volume
at first observation.

Figure 2 Stand Biomass (tonnes per ha.)

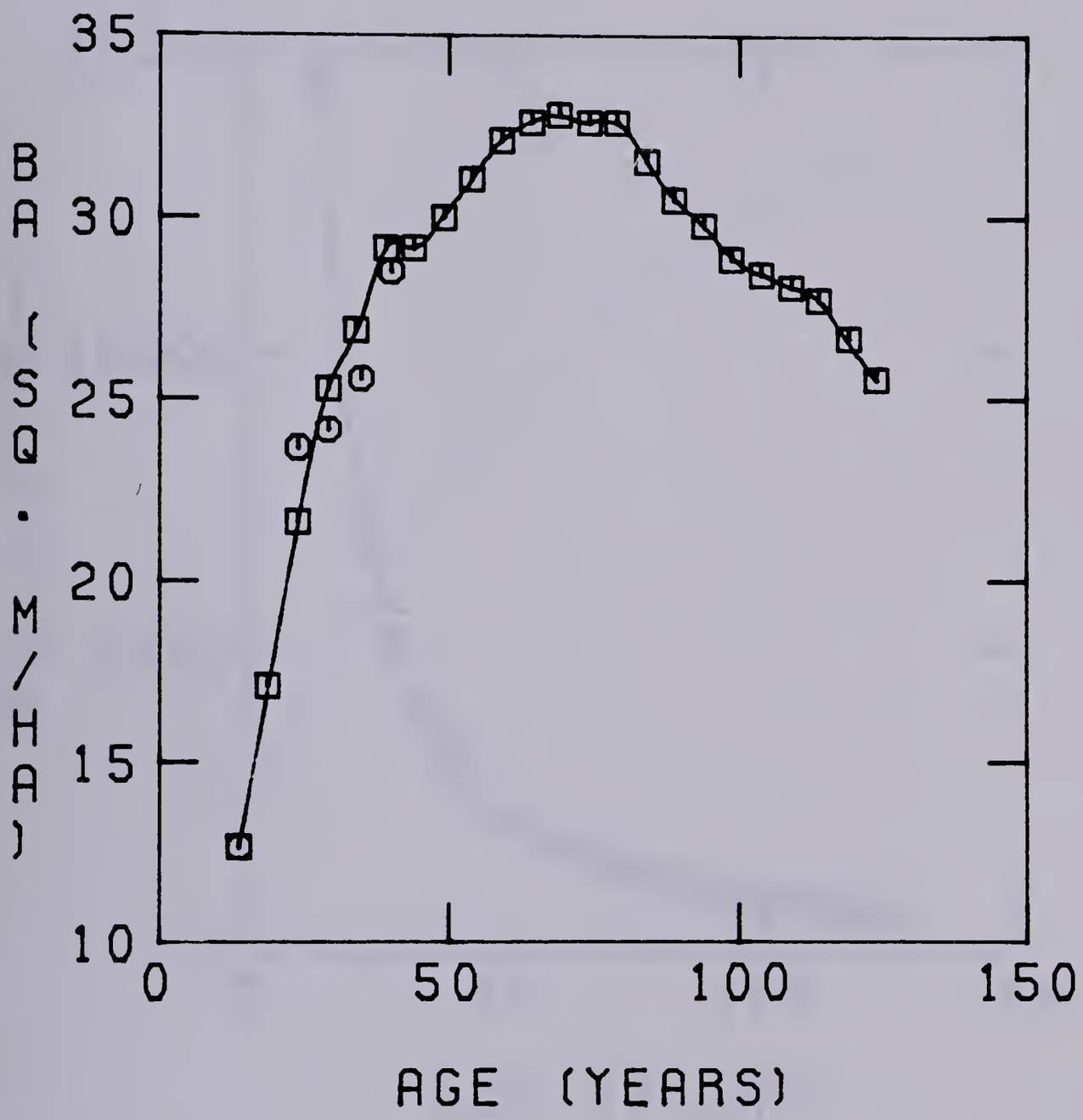


LEGEND: Actual Biomass '○'

Predicted Biomass '□'
(average of 5 runs)

NOTE: Actual Biomass = Predicted Biomass
at first observation.

Figure 3 Stand Basal Area ($m.^2$ per ha.)

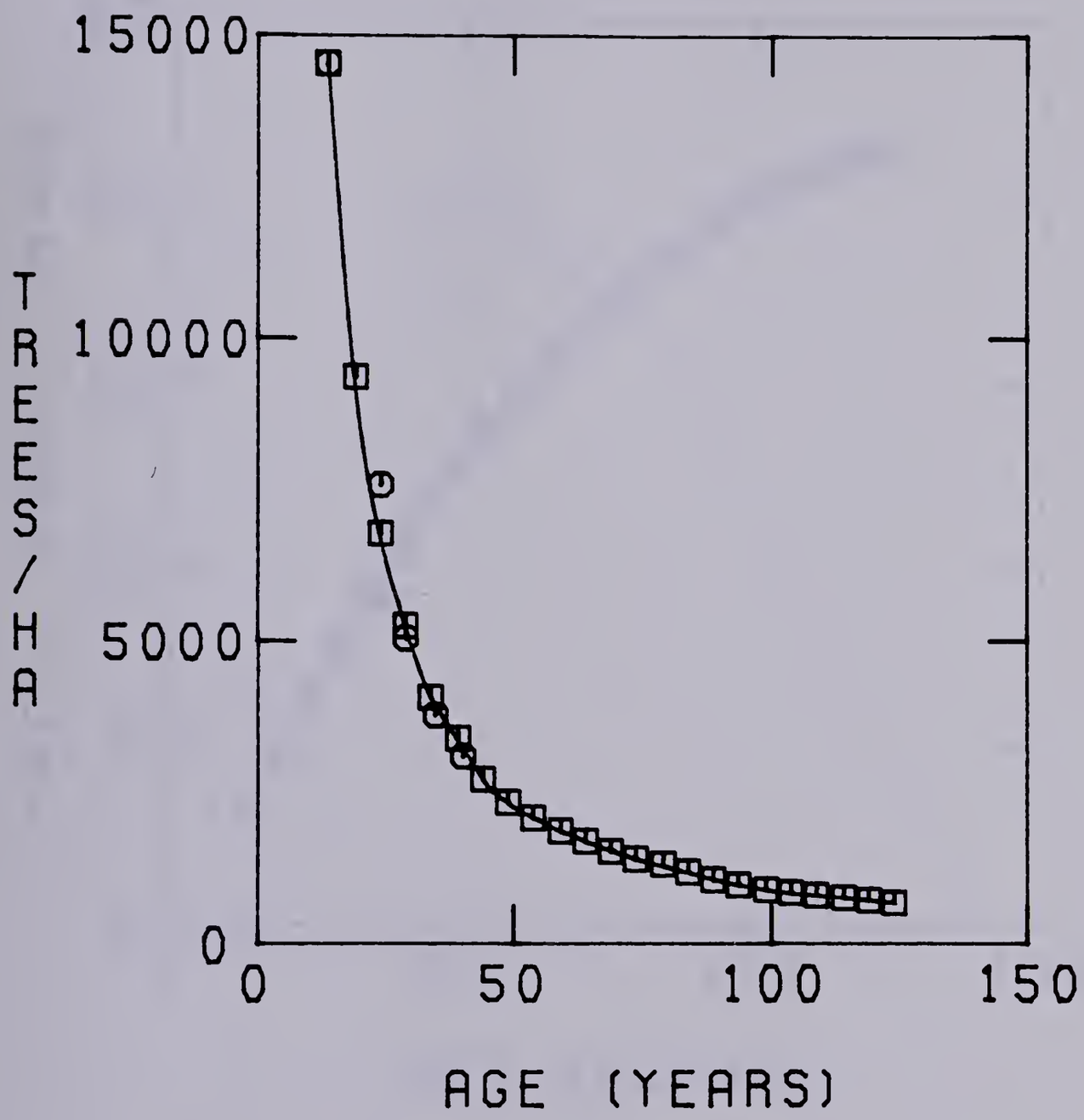


LEGEND: Actual Basal Area '○'

Predicted Basal Area '□'
(average of 5 runs)

NOTE: Actual Basal Area = Predicted Basal Area at first observation.

Figure 4 Stand Density (trees per ha.)

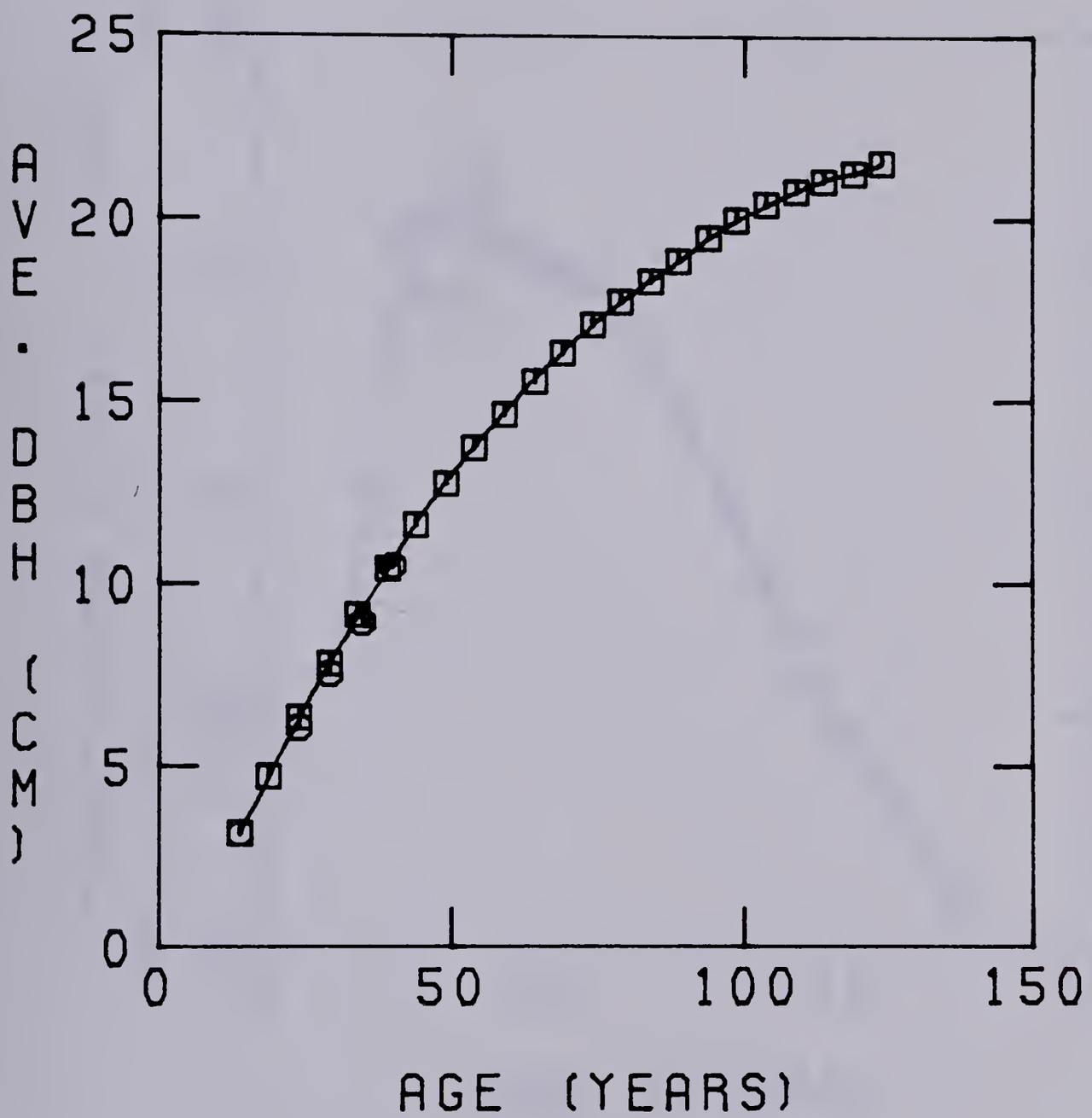


LEGEND: Actual Density '⏲'

Predicted Density '□'
(average of 5 runs)

NOTE: Actual Density = Predicted Density
at first observation.

Figure 5 Stand Average DBH (cm.)

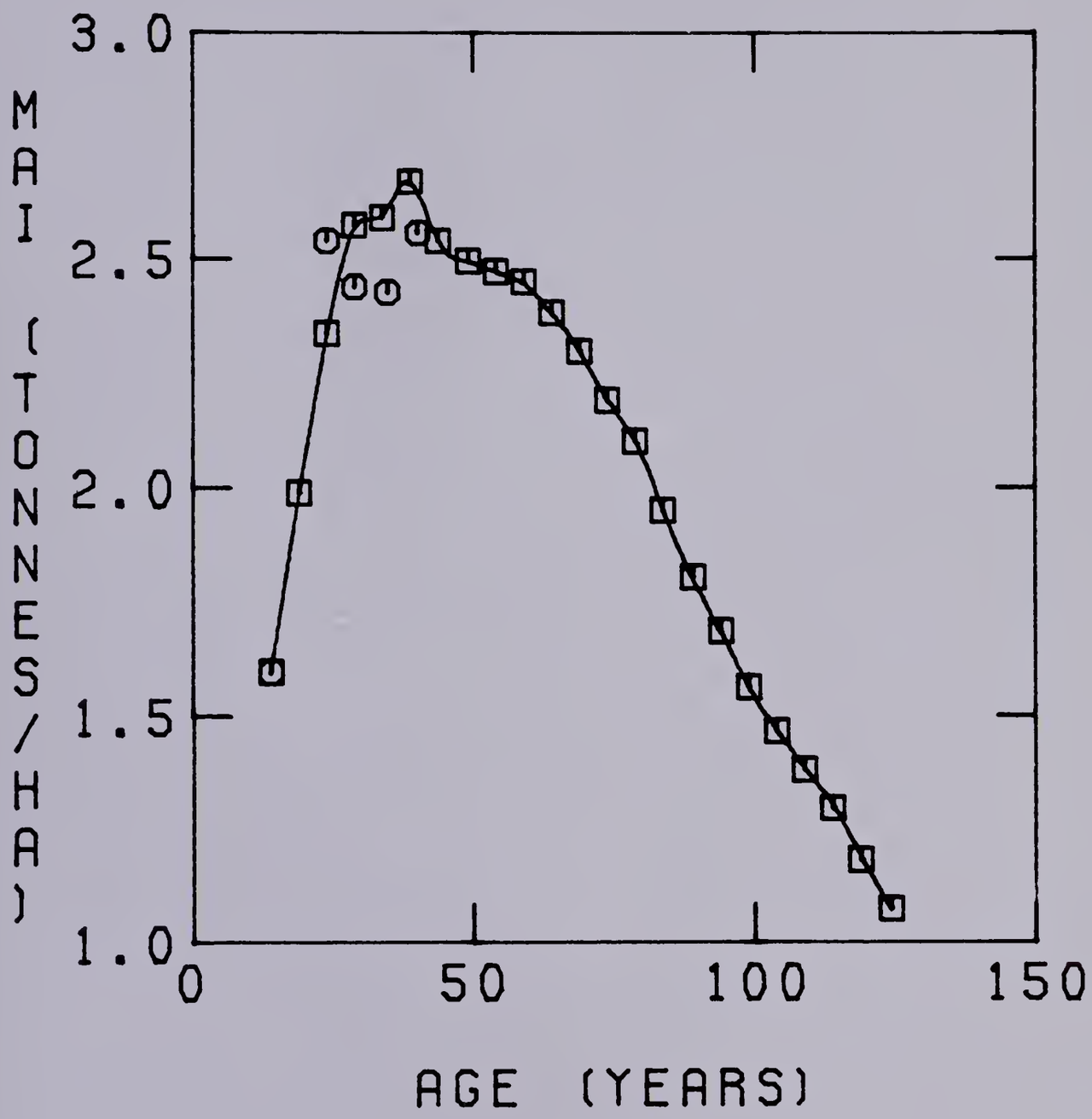


LEGEND: Actual Average DBH '○'

Predicted Average DBH '□'
(average of 5 runs)

NOTE: Actual Average DBH = Predicted Average DBH at first observation.

Figure 6 Stand Biomass Mean Annual Increment (tonnes/ha.)



LEGEND: Actual MAI 'O'

Predicted MAI (average of 5 runs) '□'

NOTE: Actual MAI = Predicted MAI at first observation.

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